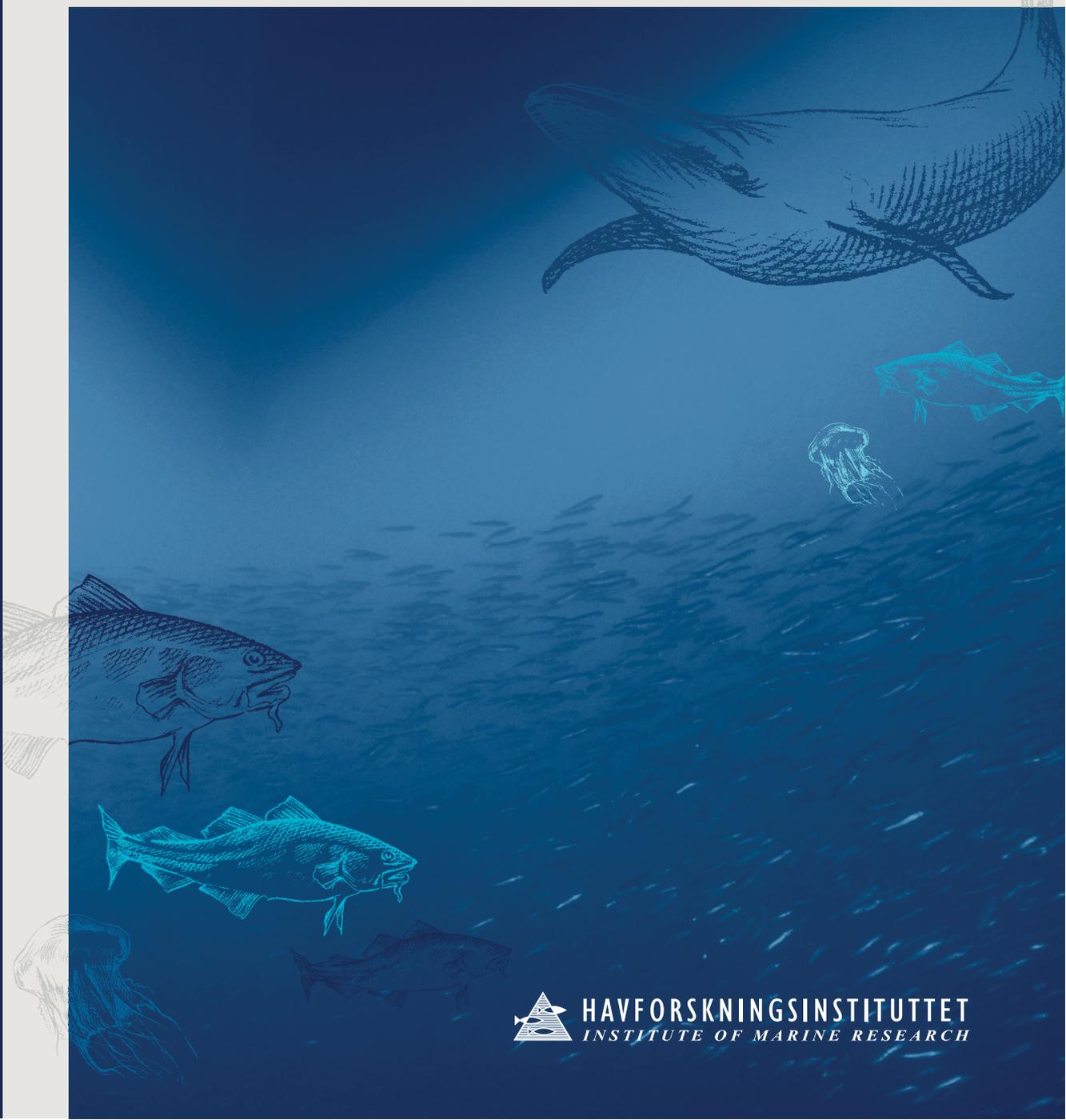


A unifying theory explaining different power budget formulations used in modern scientific echosounders for fish abundance estimation

Per Lunde and Rolf J. Korneliussen



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A unifying theory explaining different power budget formulations used in modern scientific echosounders for fish abundance estimation

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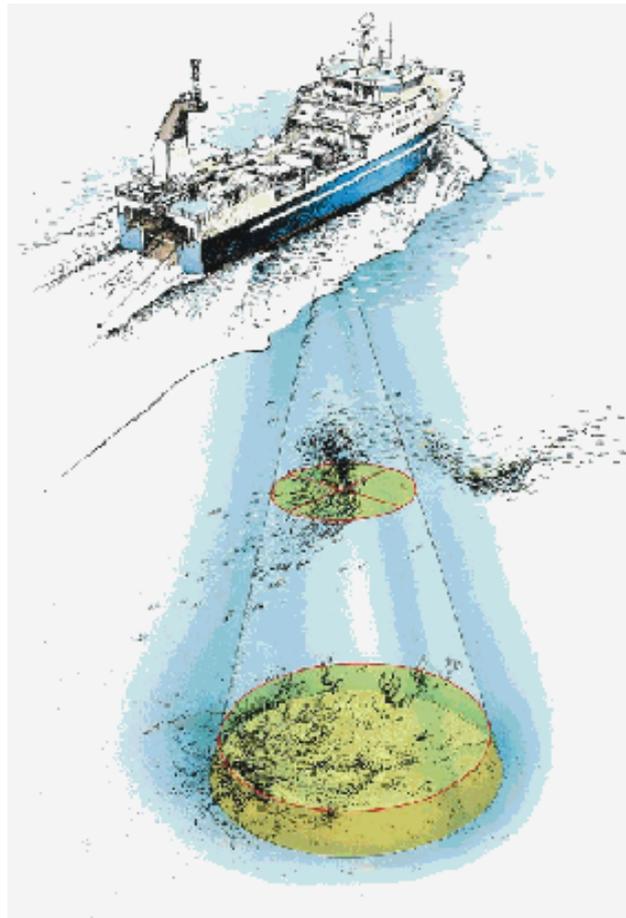


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Bergen, December 2014



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Title / Tittel: A unifying theory explaining different power budget formulations used in modern scientific echosounders for fish abundance estimation En enhetlig teori som forener ulike effektbudsjett-formuleringer brukt i moderne vitenskapelige ekkolodd for akustisk bestandsestimering av fisk		Forskningsgruppe / Research group: Marin økosystemakustikk / Marine ecosystem acoustics												
Per Lunde^{1,3} and Rolf J. Korneliussen^{2,3} ¹ University of Bergen (UoB), Dept. of Physics and Technology, Norway ² Institute of Marine Research (IMR), Bergen, Norway ³ Christian Michelsen Research AS (CMR), Bergen, Norway		Antall sider totalt / pages: 32												
Sammendrag (norsk): Akustiske metoder for mengdemåling av fisk representerer et nøkkelement innen den teknologiske basis for moderne fiskeriforskning og internasjonal regulering av marine ressurser. Akustiske metoder er basert på bruk av effektbudsjett-ligninger og kalibrerte ekkolodd-system. Ulike instrument-spesifikke formuleringer av effektbudsjett-ligninger og kalibreringsfaktorer benyttes i moderne vitenskapelige ekkolodd og sonarsystemer, så som Simrad EK500 og de nyere Simrad EK60, ES60, ME70, og MS70 - systemene. Dette er instrumenter som over de senere tiår i utstrakt grad er benyttet internasjonalt for slike anvendelser. Mangel på tilstrekkelig dokumentasjon i tidligere litteratur, vedrørende de aktuelle effektbudsjett-ligningene og kalibreringsfaktorene som benyttes i disse instrumentene, og deres sammenheng med den tradisjonelle teorien for mengdemåling av fisk, har forårsaket noe usikkerhet og forvirring blant brukere av slike systemer. Kontroll med systematiske feil og drift i slike instrumenter, forutsetter at mest mulig fullstendige matematiske funksjonsuttrykk for bestandsestimering er kjent, i form av de elektriske og akustiske parametre som benyttes i aktuelle systemer. Rapporten presenterer en enhetlig teori som ser ut til å kunne forklare de ulike (instrument-spesifikke) effektbudsjett-ligningene og kalibreringsfaktorene som benyttes i ekkolodd- og sonarsystemer. Dette inkluderer hvordan funksjonsuttrykkene som er brukt i disse instrumentene er relatert, samt deres sammenheng med den tradisjonelle og generiske (instrument-uavhengige) teorien for bestandsestimering av fisk. Inkonsistens i tidligere litteratur er forklart og korrigert. Tradisjonell teori for mengdemåling av fisk er videreutviklet og generalisert til å dekke mer fullstendige effektbudsjett-likninger for bestandsestimering og artsgjenkjennelse, ved å ivareta elektrisk terminering, representasjon av ekko-integrasjon, og de aktuelle elektriske og akustiske ekkolodd-parametre. Slik videreutvikling innbefatter både de instrument-spesifikke og de instrument-uavhengige (generiske) formuleringene. De funksjonsuttrykk som blir utledet her, utgjør en konsistent og relativt komplett teoretisk basis for forbedret forståelse og kontroll, i bruk av de konvensjonelle metoder og instrumenter for mengdemåling og artsgjenkjennelse av fisk som benyttes i dag.														

Summary (English):

Acoustic methods used in fish abundance estimation constitute a key part of the analytic assessment that makes the basis for fisheries research and international regulations of marine resources. The methods rely on power budget equations and calibrated systems. Different instrument specific formulations of power budget equations and calibration factors are used in modern scientific echosounder and sonar systems, such as the Simrad EK500 and the more recent Simrad EK60, ES60, ME70 and MS70 systems. These are instruments extensively used internationally for such applications, over the last decade or more. However, the lack of sufficient documentation in prior literature, on the actual power budget equations and calibration factors employed in these instruments, and their relationships to the traditional theory of fish abundance measurement, has caused some uncertainty and confusion among users. Control with systematic errors and drift demands the functional relationship of the abundance measurement to be fully known in terms of the electrical and acoustical parameters of the system used. The paper presents a unifying theory which seems to explain the different (instrument specific) power budget equation formulations and calibration factors employed in the mentioned echosounder and sonar systems, how they are related, and their relationship to the traditional and generic (instrument independent) theory of fish abundance measurement. Inconsistencies in prior literature are explained and corrected. Prior literature on the traditional theory is extended to provide more complete power budget equations for fish abundance estimation and species identification, by accounting for electrical termination, representation of echo integration, and the full range of electrical and acoustical echosounder parameters. These extensions apply both to the instrument specific and the generic instrument independent formulations. The expressions derived here provide a consistent and relatively complete theoretical basis for improved understanding and control in use of conventional methods and instruments for fish abundance measurement and species identification employed today.

Emneord (norsk):

- Tilbakespredning
- Volumspredning
- Ekkolodd
- Effektbudsjett
- Ekkointegrasjon
- Kalibrering
- Akustisk deteksjon av marint liv
- Bestandsestimering av fisk

Subject heading (English):

- Backscattering
- Volume scattering
- Echosounder
- Power budget equation
- Echo integrator equation
- Calibration
- Acoustical detection of marine life
- Fish abundance estimation

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Project leader (UiB)

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Abstract

Acoustic methods used in fish abundance estimation constitute a key part of the analytic assessment that makes the basis for fisheries research and international regulations of marine resources. The methods rely on power budget equations and calibrated systems. Different instrument specific formulations of power budget equations and calibration factors are used in modern scientific echosounder and sonar systems, such as the Simrad EK500 and the more recent Simrad EK60, ES60, ME70 and MS70 systems. These are instruments extensively used internationally for such applications, over the last decade or more. However, the lack of sufficient documentation in prior literature, on the actual power budget equations and calibration factors employed in these instruments, and their relationships to the traditional theory of fish abundance measurement, has caused some uncertainty and confusion among users. Control with systematic errors and drift demands the functional relationship of the abundance measurement to be fully known in terms of the electrical and acoustical parameters of the system used. The paper presents a unifying theory which seems to explain the different (instrument specific) power budget equation formulations and calibration factors employed in the mentioned echosounder and sonar systems, how they are related, and their relationship to the traditional and generic (instrument independent) theory of fish abundance measurement. Inconsistencies in prior literature are explained and corrected. Prior literature on the traditional theory is extended to provide more complete power budget equations for fish abundance estimation and species identification, by accounting for electrical termination, representation of echo integration, and the full range of electrical and acoustical echosounder parameters. These extensions apply both to the instrument specific and the generic instrument independent formulations. The expressions derived here provide a consistent and relatively complete theoretical basis for improved understanding and control in use of conventional methods and instruments for fish abundance measurement and species identification employed today.

Keywords: Echosounder, sonar, abundance estimation, fisheries acoustics, single-target backscattering, volume backscattering, power budget equation, echo integration

A unifying theory explaining different power budget formulations used in modern scientific echosounders for fish abundance estimation

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ABSTRACT

Acoustic methods used in fish abundance estimation constitute a key part of the analytic assessment that makes the basis for fisheries research and international regulations of marine resources. The methods rely on power budget equations and calibrated systems. Different instrument specific formulations of power budget equations and calibration factors are used in modern scientific echosounder and sonar systems, such as the Simrad EK500 and the more recent Simrad EK60, ES60, ME70 and MS70 systems. These are instruments extensively used internationally for such applications, over the last decade or more. However, the lack of sufficient documentation in prior literature, on the actual power budget equations and calibration factors employed in these instruments, and their relationships to the traditional theory of fish abundance measurement, has caused some uncertainty and confusion among users. Control with systematic errors and drift demands the functional relationship of the abundance measurement to be fully known in terms of the electrical and acoustical parameters of the system used. The paper presents a unifying theory which seems to explain the different (instrument specific) power budget equation formulations and calibration factors employed in the mentioned echosounder and sonar systems, how they are related, and their relationship to the traditional and generic (instrument independent) theory of fish abundance measurement. Inconsistencies in prior literature are explained and corrected. Prior literature on the traditional theory is extended to provide more complete power budget equations for fish abundance estimation and species identification, by accounting for electrical termination, representation of echo integration, and the full range of electrical and acoustical echosounder parameters. These extensions apply both to the instrument specific and the generic instrument independent formulations. The expressions derived here provide a consistent and relatively complete theoretical basis for improved understanding and control in use of conventional methods and instruments for fish abundance measurement and species identification employed today.

Keywords: Echosounder, sonar, abundance estimation, fisheries acoustics, single-target backscattering, volume backscattering, power budget equation, echo integration

1. INTRODUCTION

1.1 Acoustic fish abundance estimation

Acoustic methods are widely used for estimating fish abundance [1-5], and constitute a key part of the analytic assessment that makes the basis for international regulations of marine resources. For fish aggregated in schools or layers, echo integration [6,7] supported by biological sampling, is the most common method used in oceanic surveys [3]. The acoustic methods rely on calibrated systems [8,9] and power budget equations. Fish abundance is measured using frequencies typically in the 18-100 kHz range, whereas zooplankton measurement also employs frequencies above 100 kHz. For species identification, echosounder frequencies in the range 18 to 400 kHz or higher, are often used.

In oceanic surveys, a power budget equation for multi-target (volume) backscattering [10-14,5] is typically used to

measure volume backscattering from schools of fish, zooplankton, krill, etc. In terms of this equation, the volume backscattering coefficient s_v is measured for a sequence of thin spherical shell "ping volumes", V_p , at increasing range. The sequence of s_v measurements is integrated over the range of an observation volume V_{obs} [10,5,14], to give the target (e.g., fish) density in V_{obs} , ρ_a [15], in terms of an echo-integrator equation [7,9,3,5,14].

Prior to survey operation, a related power budget equation for single-target backscattering [10-14,5] is used for in-sea calibration of the echosounder using a metal calibration sphere [8,9,3,5], in terms of the backscattering cross section, σ_{bs} , of the sphere. The same power budget equation for single-target backscattering is used to measure the target strength, TS , of individual fish.

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1.2 Power budget equations and calibration factors used in some commonly employed echosounder systems

Different formulations of power budget equations and calibration factors are used in modern echosounder and sonar systems, such as the Simrad EK500 [10] and the more recent Simrad EK60, ES60, ME70 and MS70 [13,16-19] systems. These are instruments extensively used internationally for such applications, over the last decade or more. The expressions actually employed for abundance estimation and species identification in these instruments, are thus of importance.

It appears that the documentation in available literature, on the power budget equations and calibration factors used in these instruments, may be somewhat sparse, and in some cases insufficient. That concerns the expressions actually employed, how these are mutually related, their relationship to the traditional theory of fish abundance measurement [10], and some inconsistencies that have been revealed. These issues are outlined briefly in the following (and treated more detailed in Sections 2-9).

The traditional formulation of the power budget equations for σ_{bs} and s_v was given in the Simrad EK500 manual [10] (with more detailed and general derivations given in [11,14]). This formulation constitutes a basis for today's methods in fish abundance measurement.

In implementation of these equations in the EK500 echosounder software, two calibration factors were used, to be determined by echosounder calibration. The constants are denoted "TS transducer gain" and " S_v transducer gain" [10]. Expressions are given for these calibration factors [10], for use in calibration, but no definition appears to have been given in literature.

The mathematical relationship between the two EK500 calibration factors given in [10], and the traditional power budget equations for σ_{bs} and s_v described in [10], does not seem to be available from prior literature. No power budget equations for σ_{bs} and s_v , expressed in terms of the two EK500 calibration factors, appear to have been given. It can be shown (cf. Sections 4 and 9.3) that the expressions for the two EK500 calibration factors given in [10], cannot be readily derived from the traditional "average power" formulation of the power budget equations given in [10]. A formal representation of echo integration [14] is a prerequisite to derive the calibration factor expressions given in [10].

The EK500 manual states that "*the TS-measurement is based on the peak value of the echo samples in the sphere echo, whereas the s_A -measurement is based on integration (averaging) of the echo samples*". This difference between σ_{bs} and s_v measurements is not reflected in the power budget equations given in [10].

This situation strongly indicates that the power budget equations for σ_{bs} and s_v which are actually used in EK500, and which are presumed to be expressed in terms of the two EK500 calibration factors, may not be exactly those given in the EK500 manual [10].

For the more recent generation of Simrad scientific and fisheries echosounder and sonar systems, such as EK60, ES60, ME70 and MS70, an apparently different set of power budget equations is used [13].

Ona *et al.* [13] gave equations for σ_{bs} and s_v on logarithmic form. When these are converted to "normal" (linear) form, the equation for σ_{bs} is identical to the one given in [10]. The expression for s_v , however, differs from the equation given in [10]. In [13], a parameter $S_{a,corr}$ was used in the logarithmic expression for s_v . In the corresponding linear expression for s_v , this parameter $S_{a,corr}$ is represented in terms of a factor which is here denoted $s_{a,corr}^2$, defined by $S_{a,corr} \equiv 10\log(s_{a,corr})$. Apart from this factor $s_{a,corr}^2$, the expressions for s_v given in [13] and [10] are identical. The expression for s_v given in [13] is thus not consistent with [10]. In the fisheries research community, questions have been raised with respect to the parameter $S_{a,corr}$, in relation to what it represents, why it is introduced, and the relationship between the two different expressions for s_v given in [13] and [10].

The underlying theory for this alternative set of power budget equations [13] - including the $S_{a,corr}$ parameter - does not appear to have been documented in prior literature. As indicated above, it can be shown (cf. Sections 5 and 9.4) that the power budget equations given in [13] cannot be derived from the traditional power budget equations given in [10].

The set of power budget equations given in [13] involves two calibration factors, "Gain" and " S_a correction" [16-19]³, to be determined by echosounder calibration. In the literature available for these instruments [13,16-19], these calibration factors are not defined, nor given by any expressions.

From the above, it is very likely that the power budget equations actually used in EK60 and related instruments [16-19], may be different from those given in [13].

The unclear situation discussed above - connected to the power budget equations and the calibration factors actually used in EK500, EK60, etc., in relation to the documentation given in available literature on these instruments - has caused some uncertainty and confusion among users of these commonly employed echosounder and sonar systems.

1.3 Documentation of power budget equations and calibration factors used in different equipment

It is emphasized that, from the available literature, and practical use in fish abundance estimation [20], one has not seen any reason to question the expressions actually *implemented and used* by the manufacturer in the mentioned instruments [10,16-19]. The issue raised here, is related to the *documentation* of these expressions in available literature. As the expressions implemented have not been available for the present work, they may quite possibly be consistent with the findings and results described here.

³ In [13], these are referred to as "axial transducer gain", denoted G_0 , and "integration correction", denoted $S_{a,corr}$. Cf. also Section 9.4.

This is not the issue, - the motivation for the work is entirely principal:

- The equations used for international regulations of marine resources, including their theoretical basis, should preferably be known and documented in available literature.
- Different formulations of the power budget equations and calibration factors used, in different equipment, should preferably be mutually consistent, and consistent with the traditional theory of abundance estimation [10].
- An improved documentation with respect to the expressions and calibration factors actually employed in different equipment, the relationship between these, and their relationships to the traditional theory of fish abundance estimation, may contribute to reduce uncertainty and confusion among users, and further improve confidence in such equipment.
- The power budget equations forming the basis for abundance measurement, should be sufficiently complete to enable accurate and reliable analyses of error and system drift. The expressions given in [10] and elsewhere [12,13,5] do not account for electrical termination and formal representation of echo integration.

Such aspects may be particularly important these days, as new multi-frequency echosounders, sonars and broadband measurement methods are developed and introduced in the market [21]. Measurement and calibration methods using these new broadband instruments should preferably be based on a precise, consistent and documented theory.

1.4 Completeness of conventional power budget equations

One might argue that since scientific echosounders used for fisheries abundance estimation and species identification are being calibrated, a full functional relationship – giving the expressions actually used for the abundance measurement – is not needed. Use of different instrument specific power budget formulations, combined with unspecified calibration factors which are solely determined empirically in echosounder calibration, such as “ TS transducer gain” and “ S_v transducer gain” for EK500, and “Gain” and “ S_a correction” for EK60, etc., may indeed be sufficient for many applications.

However, there are situations where documentation of a consistent and more complete functional relationship for the abundance measurement is required. This is the case e.g. for investigation of, and possible correction for, measurement errors due to system drift, caused by environmental changes from echosounder calibration to oceanic surveying, or other effects.

For example, effects of sea water temperature may influence on the echosounder system performance, and compensation for system drift caused by the temperature dependencies of the system, may possibly be needed. A shift in the echosounder system gain may result from shifts in the transducer’s transmit and receive frequency responses, its input electrical impedance, and its beam pattern, due to changes in sea temperature. Since it is standard practice to calibrate echosounder systems for fishery surveys in one environment (typically a sheltered area), and apply the resulting gains to interpret data collected over the

range of sea temperatures encountered during a survey, the resulting fish abundance estimates may be biased [12].

For reliable analysis of such situations, for a given echosounder, the equations actually used for the abundance measurement in that echosounder, need to be fully known, and expressed in terms of the calibration factors involved. Also, the calibration factors themselves need to be fully known in terms of the range of echosounder system parameters. That is, the properties of the transducer, electronics, sea water, electrical signal, echo integration method, acoustic sound field, etc.

1.5 Objectives and outline

A primary objective of the present work is to derive a unifying theory which – on basis of the traditional and generic (instrument independent) theory of fish abundance measurement, and use of a single and consistent terminology – may be able to

- (a) explain the different power budget equation formulations and calibration factors employed in the Simrad EK500, EK60, ES60, ME70 and MS70 systems,
- (b) explain how these different instrument dependent formulations are mutually related,
- (c) explain how they are related to the traditional and generic (instrument independent) theory of fish abundance measurement, such as given in the EK500 manual [10],
- (d) resolve possible inconsistencies in prior literature on these instruments,
- (e) provide definitions of, relationships between, and explanation for use of, the various calibration factors “ TS transducer gain” and “ S_v transducer gain” (for EK500), “Gain” and “ S_a correction” (for EK60, etc.), and
- (f) explain how these are mathematically related to the power budget equations of the respective instruments.

A second objective is to generalize the traditional and generic power budget equations for σ_{bs} and s_v , that are used in conventional fish abundance measurement and species identification [10], to account for arbitrary electrical termination, representation of echo integration, and the full range of electrical and acoustical echosounder parameters. The purpose is to provide a basis for improved analysis of measurement errors and system drift, for the instrument specific as well as the generic (instrument independent) formulations.

The paper is organized as follows. Generic power budget equations for σ_{bs} and s_v , formulated on average power and echo-integration forms [14], are summarized in Sections 2 and 3, and denoted “formulation A” and “B”, respectively. These instrument independent equations serve as a basis for the subsequent analysis. In Section 4 and 5, two alternative and instrument specific formulations of the power budget equations are derived, denoted “formulation C” and “D”, where these are shown to correspond to, and generalize, the power budget equations and calibration factors implemented in the Simrad EK500, and the Simrad EK60 and related echosounder/sonar systems, respectively. In Section 6, an alternative “formulation E” of the power budget equations is derived, to explore the consequences of an interpretation indicated in ref. [13]. A generic formulation of the power budget equations covering all echo integration

formulations B-E is presented in Section 7, particularly suitable for analysis of error and system drift. Based on formulations A-E, an echo integrator equation for fish density and biomass estimation is derived in Section 8. The various formulations A-E are discussed in more detail in Section 9, and related to the power budget equations and calibration factors used conventionally [10,13,16-19]. Conclusions are given in Section 10. For convenience, Appendix A gives an overview of the equations involved in formulations A-E. An interpretation of the power budget equations in terms of power flow is given in Appendix B.

For completeness, the power budget equations for σ_{bs} and s_v , and the calibration factors involved in the various formulations A-E described in Sections 2-6, are given both on “normal” (linear) and logarithmic (dB) forms, since both forms are commonly used in the literature of acoustic fishery research (either the former or the latter, usually). There may also be a need to correct some equations on logarithmic (dB) form which are ill-expressed in this literature (involving dimensional arguments of the \log function). For convenience, power budget equations and calibration factors belonging to the main results of the work, are framed.

In the work presented here, the theory is treated on a relative generic level, in relation to signal processing implementation. Different solutions may be chosen, e.g. with respect to methods for calculation of echo integrals. Such aspect are not addressed here.

The unifying theory developed and presented here, appears to be able to explain and resolve the questions and unresolved issues that are addressed in Sections 1.2-1.4.

2. GENERIC POWER BUDGET EQUATIONS, - FORMULATION A

2.1 Assumptions

The analysis is based on the following assumptions for the echosounder and its environment (cf. [14] and references therein):

- (a) The monostatically operated transducer is passive and reciprocal;
- (b) the transmit voltage amplitude is sufficiently small to avoid nonlinear effects in the electroacoustic transducer and electronics (i.e., the transducer and electronics are operated in their linear ranges);
- (c) the electrical impedances of the transducer and receiving electronics are approximately constant in the narrow frequency band of a sonar ping;
- (d) the fluid medium (seawater) is homogeneous, with constant density and sound velocity;
- (e) the amplitudes of the transmitted sound pressure signals are sufficiently small so that finite-amplitude sound propagation effects in seawater can be neglected;
- (f) targets are in the far field of the transducer;
- (g) possible nonlinear effects in the scattering process at the target itself (involving e.g. fish with gas-filled swim-

bladder), can be neglected, so that linear backscattering theory applies;

- (h) the volume backscattering coefficient can be calculated as a sum of backscattering cross sections (i.e., intensities) per unit volume;
- (i) the scattering objects are uniformly distributed in the observation volume, with
- (j) random phases of the scattered echoes (i.e., random spacing of scattering objects, and movement of objects from one transmission to the next);
- (k) possible multiple-scattering effects and interaction between objects are neglected;
- (l) excess attenuation from power extinction caused by volume scattering is neglected; and
- (m) the same transmit electrical power Π_T is used for σ_{bs} and s_v measurements, i.e., in calibration and surveying.

These are all common assumptions underlying the traditional theory of fish abundance measurement [1-14]. The discussion of their validity is an extensive and complex subject, beyond the scope of the present work, and discussed elsewhere. Assumptions (a)-(c) relate to the transducer and electric components of the echosounder system, and are normally fulfilled by driving the piezoelectric transducer using low electrical power. In relation to (d): in abundance estimation, the sound velocity is typically taken to be the average value of the sound velocity profile, over the depth range in question [21]. Assumption (e) is addressed by refs. [11,21,22,23], and maximum electrical transmit powers have been suggested [21]. Assumption (g) is discussed e.g. in ref. [24]. The assumptions (h)-(l) are included in the set of assumptions used by Clay and Medwin [1,4] to derive the analogous “in-water” expressions for s_v , accounting for acoustic pressures in the sea only. Relatively extensive discussions on the validity of (h)-(l) are given by refs. [1,4,5,28], also summarizing other literature addressing these issues.

A time harmonic factor $e^{i\omega t}$ is assumed and suppressed, where $i = \sqrt{-1}$, $\omega = 2\pi f$ is the angular frequency of the harmonic wave, f is the frequency, and t is the time. Bold-face letters are used to indicate complex-valued quantities.

A spherical coordinate system is used, with coordinates (r, θ, φ) , origin at the centre of the transducer front, and with the z axis (i.e. $\theta = \varphi = 0$) chosen normal to the transducer’s front surface, and assumed coincident with the transducer’s acoustical beam axis [14]. r is the radial distance, denoted range, θ is the polar angle (rel. to the z axis), and φ is the azimuthal angle (rel. to the x axis).

In fisheries acoustics, another coordinate system is often used, (r, α, β) , where r is the range as above, and α and β are orthogonal “alongship” and “athwartship” angles in the x - z and y - z planes, respectively, both referred to the z axis. The x and y axes correspond to the “alongship” and “athwartship” directions, respectively. The transformations between the two coordinate systems are $\tan \alpha = \tan \theta \cos \varphi$, $\tan \beta = \tan \theta \sin \varphi$; and $\tan \theta = (\tan^2 \alpha + \tan^2 \beta)^{1/2}$, $\sin \varphi = (1 + (\tan \alpha / \tan \beta)^2)^{-1/2}$, respectively.

2.2 Average power formulation

Under the above assumptions, it can be shown from basic acoustic principles that the backscattering cross section of a single scattering target located at position (r, θ, φ) in the transducer's far field, σ_{bs} (m²), and the volume backscattering coefficient for a thin spherical shell "ping volume" V_p in the far field, s_v (m⁻¹), are given as [11,14]

$$\sigma_{bs} = \frac{16\pi^2 \cdot r^4 \cdot e^{4\alpha r} \cdot \Pi_R^{st}}{G^2(\theta, \varphi) \cdot \lambda^2 \cdot F_{\Pi} \cdot \Pi_T}, \quad (1)$$

$$s_v = \frac{32\pi^2 \cdot r^2 \cdot e^{4\alpha r} \cdot \Pi_R^v}{G_0^2 \cdot \psi \cdot \lambda^2 c_0 \cdot \tau_p \cdot F_{\Pi} \cdot \Pi_T}, \quad (2)$$

respectively. Here, τ_p is the duration (s) of the transmitted voltage signal (denoted "ping duration"). In Eq. (1), r is the distance to the single target (m). In Eq. (2), $r \approx r_p \equiv (r_{p1} + r_{p2})/2$ is the mid-range of the "ping volume" V_p (m³), that is contained within ranges r_{p1} and r_{p2} , with thickness $dr_p \equiv r_{p2} - r_{p1} = \frac{1}{2} c_0 \tau_p$ [14]. α is the sound pressure acoustic absorption coefficient of seawater (expressed in Np/m). $\lambda = c_0/f$ is the acoustic wavelength (m), where c_0 is the small-signal sound velocity in seawater (m/s).

$G(\theta, \varphi)$ and G_0 (both non-dimensional) are the transducer gain and the axial transducer gain, defined⁴ as [25, 10,11,14]

$$\begin{aligned} G(\theta, \varphi) &\equiv \eta \cdot D(\theta, \varphi) \\ &= \eta \cdot \frac{4\pi |\mathbf{B}_i(\theta, \varphi)|^2}{\int_{4\pi} |\mathbf{B}_i(\theta, \varphi)|^2 d\Omega} = G_0 \cdot |\mathbf{B}_i(\theta, \varphi)|^2, \end{aligned} \quad (3)$$

$$G_0 \equiv G(0,0) = \eta \cdot D_0, \quad (4)$$

respectively, where η is the transducer's electroacoustic conversion efficiency (the non-dimensional ratio of radiated acoustic to transmitted electrical power), and

$$D_0 = \frac{4\pi}{\int_{4\pi} |\mathbf{B}_i(\theta, \varphi)|^2 d\Omega}, \quad (5a)$$

$$D(\theta, \varphi) = \frac{4\pi \cdot |\mathbf{B}_i(\theta, \varphi)|^2}{\int_{4\pi} |\mathbf{B}_i(\theta, \varphi)|^2 d\Omega} = D_0 \cdot |\mathbf{B}_i(\theta, \varphi)|^2, \quad (5b)$$

and $\mathbf{B}_i(\theta, \varphi)$, are the axial directivity factor [26], the directivity factor [25], and the beam pattern (the angular distribution of

the sound pressure, normalized to the axial sound pressure) [26], respectively, for the transmitted sound pressure field.

$D_0 \equiv D(0,0)$ and $D(\theta, \varphi)$ represent the (non-dimensional) ratios of the transducer's axial intensity, respectively the intensity in a given direction (θ, φ) , to the intensity of an omnidirectional (point) source generating the same acoustic power [26]. $G(\theta, \varphi)$ describes how much acoustic power is radiated in the (θ, φ) direction, under conditions of small-amplitude and lossless sound propagation, relative to that of an omnidirectional (point) source, which is radiating (acoustically) the amount of electrical power supplied to the transducer, [25,14].

ψ (in steradians, sr) is the equivalent two-way beam solid angle of the transducer, defined by [1,9,10,4,25,14]

$$\psi \equiv \int_{4\pi} |\mathbf{B}_i(\theta, \varphi)|^4 d\Omega = \frac{1}{G_0^2} \int_{4\pi} G^2(\theta, \varphi) d\Omega \quad (6)$$

as the solid angle through which all the acoustic power would flow if the two-way radiation intensity was constant (and equal to the maximum value of that intensity) for all angles within ψ [25]. ψ thus represents an effective beam width of the transducer's intensity field, expressed in terms of this solid angle, accounting for the combined effect of transmission and reception.

The average transmitted and received electrical powers at the transducer terminals (W) are given as [26,14]

$$\Pi_T = \frac{|\mathbf{V}_T|^2 R_T}{2|\mathbf{Z}_T|^2} = \frac{R_T}{|\mathbf{Z}_T|^2} \cdot (V_T^{rms})^2, \quad (7a)$$

$$\Pi_R^{st} = \frac{|\mathbf{V}_R^{st}|^2 R_E}{2|\mathbf{Z}_E|^2} = \frac{R_E}{|\mathbf{Z}_E|^2} \cdot (V_R^{st,rms})^2, \quad (7b)$$

$$\Pi_R^v = \frac{|\mathbf{V}_R^v|^2 R_E}{2|\mathbf{Z}_E|^2} = \frac{R_E}{|\mathbf{Z}_E|^2} \cdot (V_R^{v,rms})^2, \quad (7c)$$

respectively, where "average" refers to averaging over one cycle of the harmonic signal waveform, at the frequency f . V_T , V_R^{st} and V_R^v are the amplitudes of the voltage signals (V) across the transducer's electrical terminals during transmission and reception, i.e., for the transmitted signal and the single-target and volume backscattered echoes, respectively. $V_T^{rms} = |V_T|/\sqrt{2}$, $V_R^{st,rms} = |V_R^{st}|/\sqrt{2}$ and $V_R^{v,rms} = |V_R^v|/\sqrt{2}$ are the effective (rms) amplitudes of the transmitted and received voltage waveforms, $V_T(t)$, $V_R^{st}(t)$ and $V_R^v(t)$.

Finally,

$$F_{\Pi} \equiv \frac{4R_T R_E}{|\mathbf{Z}_R + \mathbf{Z}_E|^2} \quad (8)$$

is a (non-dimensional) electrical termination factor for the power budget equations formulated in terms of average power

⁴ In acoustics, "transducer gain" corresponds to "antenna gain" used in electromagnetic communication and radar theory, cf. e.g. [25].

[11,14]. F_{Π} accounts for arbitrary electrical termination at signal reception. Here, $\mathbf{Z}_T = R_T + iX_T$ is the transducer's input electrical impedance (ohm) at transmission, when radiating into the fluid, with resistance and reactance R_T and X_T , respectively. $\mathbf{Z}_R = R_R + iX_R$ is the output (internal) electrical impedance of the receiving transducer, and $\mathbf{Z}_E = R_E + iX_E$ is the input electrical impedance of the receiving electric network ("electrical termination impedance"), involving resistances and reactances R_R and X_R , and R_E and X_E , respectively.

Eqs. (1)-(2) are equal to the expressions given in the EK500 manual [10], except for the factor F_{Π} which was neglected there, implicitly implying $F_{\Pi} = 1$ [11,14]. $F_{\Pi} = 1$ corresponds to the particular case of conjugate matched electrical termination, $\mathbf{Z}_E = \mathbf{Z}_R^*$ (where "*" denotes complex conjugation); or $\mathbf{Z}_E = \mathbf{Z}_R$ and $X_T = 0$ (which is valid in a frequency band close to the series resonance frequency of the transducer vibration mode used) [14].

Dividing σ_{bs} by the a reference area r_1^2 (chosen equal to 1 m²), multiplying s_v by a reference length r_2 (chosen equal to 1 m), and applying 10log on both sides of Eqs. (1) and (2), leads to the logarithmic (dB) versions of Eqs. (1)-(2),

$$TS = 10\log\left(\frac{\Pi_R^{st}}{\Pi_T}\right) + 40\log\left(\frac{r}{r_3}\right) + 2\hat{\alpha}r + 10\log\left(\frac{16\pi^2 \cdot r^4}{\lambda^2 \cdot r_1^2}\right) - 20\log(G_0) - 40\log|\mathbf{B}_i(\theta, \varphi)| - 10\log(F_{\Pi}), \quad (9)$$

$$S_v = 10\log\left(\frac{\Pi_R^v}{\Pi_T}\right) + 20\log\left(\frac{r}{r_3}\right) + 2\hat{\alpha}r + 10\log\left(\frac{32\pi^2 \cdot r_2 \cdot r_3^2}{\lambda^2 \cdot c_0 \cdot \tau_p}\right) - 20\log(G_0) - 10\log(\psi) - 10\log(F_{\Pi}), \quad (10)$$

where [15] $TS \equiv 10\log(\sigma_{bs}/r_1^2)$ (dB re. r_1^2) is the target strength of a single scattering target; $S_v \equiv 10\log(s_v r_2)$ (dB re. r_2^{-1}) is the volume backscattering strength for the "ping volume" V_p ; $\hat{\alpha} \equiv \alpha \cdot 20\log e \approx 8.686\alpha$ is the sound pressure absorption coefficient of seawater expressed in dB/m; r_3 is a reference length for r (chosen equal to 1 m); and Eq. (3) has been used.

Eqs. (1)-(2), or alternatively Eqs. (9)-(10), constitute formulation A of the power budget equations for σ_{bs} and s_v , on "normal" and logarithmic (dB) forms, respectively.

Physical interpretations of Eqs. (1)-(2) in terms of power flow are given in Appendix B.

3. GENERIC ECHO INTEGRATION POWER BUDGET EQUATIONS, - FORMULATION B

Calculation of the electrical powers Π_T , Π_R^{st} and Π_R^v involved in the average power formulation of the power budget equa-

tions, Eqs. (1)-(2), is normally made using time integration of voltage signals, denoted echo integration [3,5,6,7]. An echo integration formulation of the power budget equations for σ_{bs} and s_v (denoted "formulation B") is given in the following, derived from Eqs. (1)-(2).

3.1 Generic echo integration formulation

In formulation B, σ_{bs} and s_v are equivalently to Eqs. (1)-(2) given as [14]

$$\sigma_{bs} = \frac{16\pi^2 \cdot r^4 \cdot e^{4\alpha r} \cdot \tau_p \cdot [tivs]_R^{st}}{G^2(\theta, \varphi) \cdot \lambda^2 \cdot \tau_{st} \cdot F_{VV}^2 \cdot [tivs]_T}, \quad (11)$$

$$s_v = \frac{32\pi^2 \cdot r^2 \cdot e^{4\alpha r} \cdot [tivs]_R^{sv}}{G_0^2 \cdot \psi \cdot \lambda^2 \cdot c_0 \cdot \tau_g \cdot F_{VV}^2 \cdot [tivs]_T}, \quad (12)$$

where

$$[tivs]_T \equiv \int_0^{\tau_p} |V_T(t)|^2 dt, \quad (13)$$

$$[tivs]_R^{st} \equiv \int_{t_{st1}}^{t_{st2}} |V_R^{st}(t)|^2 dt, \quad (14a)$$

$$[tivs]_R^{sph} \equiv \int_{t_{s1}}^{t_{s2}} |V_R^{sph}(t)|^2 dt, \quad (14b)$$

$$[tivs]_R^{sv} \equiv \int_{t_{g1}}^{t_{g2}} |V_R^{sv}(t)|^2 dt, \quad (14c)$$

are defined as the "echo integral" (in unit of V²s = W·ohm·s) – or "time-integral-voltage-squared" $[tivs]$ – values of the transmitted voltage pulse (the "ping"), $V_T(t)$, and three different received voltage waveforms, $V_R^{st}(t)$, $V_R^{sph}(t)$ and $V_R^{sv}(t)$, for a single target echo (e.g., individual fish), a calibration sphere echo, and the echo received from a "gated volume" in oceanic surveying, respectively [14]. From Eqs. (7) and (13)-(14), these are related to the average transmitted and received electrical powers by [14]

$$\Pi_T \approx \frac{R_T}{|\mathbf{Z}_T|^2} \cdot \frac{1}{\tau_p} \int_0^{\tau_p} |V_T(t)|^2 dt = \frac{R_T}{|\mathbf{Z}_T|^2 \tau_p} \cdot [tivs]_T, \quad (15)$$

$$\Pi_R^{st} \approx \frac{R_E}{|\mathbf{Z}_E|^2} \cdot \frac{1}{\tau_{st}} \int_{t_{st1}}^{t_{st2}} |V_R^{st}(t)|^2 dt = \frac{R_E}{|\mathbf{Z}_E|^2 \tau_{st}} \cdot [tivs]_R^{st}, \quad (16)$$

$$\Pi_R^{sph} \approx \frac{R_E}{|\mathbf{Z}_E|^2} \cdot \frac{1}{\tau_s} \int_{t_{s1}}^{t_{s2}} |V_R^{sph}(t)|^2 dt = \frac{R_E}{|\mathbf{Z}_E|^2 \tau_s} \cdot [tivs]_R^{sph}, \quad (17)$$

$$\Pi_R^{sv} \approx \frac{R_E}{|\mathbf{Z}_E|^2} \cdot \frac{1}{\tau_g} \int_{t_{g1}}^{t_{g2}} |V_R^{sv}(t)|^2 dt = \frac{R_E}{|\mathbf{Z}_E|^2 \tau_g} \cdot [tivs]_R^{sv}, \quad (18)$$

where Π_R^{st} , Π_R^{sph} and Π_R^{sv} are the received electrical powers - averaged over one cycle of the harmonic waveform - of a sin-

gle-target echo, a calibration sphere echo, and for volume scattering from the gated volume, respectively.

$\tau_{st} \equiv t_{st2} - t_{st1}$ is the time duration (s) of the voltage echo $V_R^{st}(t)$, received from a single scattering target, where t_{st1} and t_{st2} are the arrival times of the start and end of $V_R^{st}(t)$.

Similarly, $\tau_s \equiv t_{s2} - t_{s1}$ is the time duration of the voltage echo $V_R^{sph}(t)$, received from the calibration sphere, where t_{s1} and t_{s2} are the arrival times of the start and end of $V_R^{sph}(t)$. τ_s is always longer than τ_p , due to “ringing” (transient decay) caused by limited transducer bandwidth, the finite dimensions of the sphere, possible “ringing” due to elastic sphere vibration modes and circumferential waves at the sphere surface, etc., in the sphere echo.

$\tau_g \equiv t_{g2} - t_{g1}$ is the “gate opening time” (s) used for the voltage waveform $V_R^{gv}(t)$, received from the “ping volume” V_p , where t_{g1} and t_{g2} are the times of gate opening and closure [14]. In general, τ_g may be smaller, equal to, or larger than τ_p , depending on the echo integration solutions implemented in the echosounder system [1,4,14]. A choice $\tau_g = \tau_p$ may often be relevant in practice, such as to avoid counting fish twice in the calculation of ρ_a , cf. Eqs. (76) and (78). This appears to have been used e.g. for the expressions given in [13].

For completeness, to avoid mixing the concepts of “gate opening time” and “ping duration” (cf. e.g. Section 9.5), the following description covers the more general situations where τ_g can be chosen independently of τ_p (including $\tau_g = \tau_p$). That is, if the “gate opening time” equals the “ping duration”, so that $\tau_g = \tau_p$, one may in the present work replace τ_g by τ_p in all expressions involving τ_g .

Finally,

$$F_{VV} \equiv \frac{2R_T |Z_E|}{|Z_R + Z_E| |Z_T|}, \quad (19)$$

is the (non-dimensional) electrical termination factor for the power budget equations formulation in terms of echo integration. F_{VV} accounts for arbitrary electrical termination at signal reception [14].

Note that if F_{VV} is neglected in Eqs. (11)-(12), that corresponds to setting $F_{VV} = 1$. $F_{VV} = 1$ corresponds to the particular case of conjugate matched electrical termination, $Z_E = Z_R^*$, when $X_T = 0$; or $Z_E = Z_R$ when $X_T = 0$. The condition $X_T = 0$ applies to a frequency band close to the series resonance frequency of the transducer vibration mode used [14].

The logarithmic versions of Eqs. (11)-(12), giving the target strength of a single target, TS , and the volume backscattering strength, S_v , are given as

$$TS = 10 \log \left(\frac{[tivs]_R^{st}}{[tivs]_T} \right) + 40 \log \left(\frac{r}{r_3} \right) + 2\hat{\alpha}r + 10 \log \left(\frac{16\pi^2 \cdot r_3^4}{\lambda^2 \cdot r_1^2} \right) - 20 \log(G_0) - 40 \log |B_i(\theta, \varphi)| + 10 \log \left(\frac{\tau_p}{\tau_{st}} \right) - 20 \log(F_{VV}), \quad (20)$$

$$S_v = 10 \log \left(\frac{[tivs]_R^{gv}}{[tivs]_T} \right) + 20 \log \left(\frac{r}{r_3} \right) + 2\hat{\alpha}r + 10 \log \left(\frac{32\pi^2 \cdot r_2 \cdot r_3^2}{\lambda^2 c_0 \cdot \tau_g} \right) - 20 \log(G_0) - 10 \log(\psi) - 20 \log(F_{VV}), \quad (21)$$

where Eq. (3) is used.

Eqs. (11)-(12), or alternatively Eqs. (20)-(21), constitute formulation B of the power budget equations for σ_{bs} and s_v , on “normal” and logarithmic (dB) forms, respectively.

Since σ_{bs} and s_v are both given in terms of G_0 [cf. Eq. (3)], formulation B involves use of a single calibration factor, G_0 . In the following, expressions are derived for this calibration factors. Two approaches are used. The first approach (denoted “method 1”) gives the formulation B calibration factors G_0 in terms of the echosounder parameters that are measured at calibration. The second approach (denoted “method 2”) is used to (a) establish alternative relationships for use in calibration, and (b) express G_0 on a form that is similar to forms used in the EK500 manual [10] for the two calibration factors of the EK500 system, cf. Eqs. (38) and (45).

3.2 Formulation B calibration factor G_0

3.2.1 Method 1

Consider a calibration situation, with the centre of the calibration sphere located at position $(r_s, \theta_s, \varphi_s)$ ⁵. From Eqs. (11) and (3), G_0 can be determined from calibration measurement data as

$$G_0^2 = \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\hat{\alpha}r_s} \cdot \tau_p \cdot [tivs]_R^{sph}}{\lambda_s^2 \cdot F_{VV,s}^2 \cdot \tau_s \cdot \sigma_{bs,theory}^{sph} \cdot |B_i(\theta_s, \varphi_s)|^4 \cdot [tivs]_T}, \quad (22)$$

where $\sigma_{bs,theory}^{sph}$ is a theoretically known (calculated) value for the backscattering cross section of the sphere, at the frequency in question. $F_{VV,s}$, $\lambda_s \equiv c_{0s}/f$ and c_{0s} are the electrical termination factor, the acoustic wavelength, and the sound velocity, in seawater at calibration conditions. The logarithmic version of Eq. (22) is

⁵ Parameters which are measured or assumed known at calibration using the metal calibration sphere, are given a subscript “s”.

$$\begin{aligned}
20\log(G_0) &= 10\log\left(\frac{[tivs]_R^{sph}}{[tivs]_r}\right) + 40\log\left(\frac{r_s}{r_3}\right) + 2\hat{\alpha}r_s \\
&+ 10\log\left(\frac{16\pi^2 \cdot r_3^4}{\lambda_s^2 \cdot r_1^2}\right) - TS_{theory}^{sph} - 40\log|\mathbf{B}_i(\theta_s, \varphi_s)|, \quad (23) \\
&+ 10\log\left(\frac{\tau_p}{\tau_s}\right) - 20\log(F_{VV,s})
\end{aligned}$$

where $TS_{theory}^{sph} \equiv 10\log(\sigma_{bs,theory}^{sph}/r_1^2)$ is a theoretically known (calculated) value for TS of the calibration sphere.

Quantities to be measured in echosounder calibration for determination of the calibration factor G_0 are $r_s, \theta_s, \varphi_s, \tau_p, \tau_s, [tivs]_R^{sph}, [tivs]_r, |\mathbf{B}_i(\theta_s, \varphi_s)|$ and c_{0s} . $F_{VV,s}$ and $\sigma_{bs,theory}^{sph}$ are typically assumed to be known. In practice, calibration measurement of G_0 (or alternative calibration factors, cf. Sections 4-6) may be done for several sphere positions $(r_s, \theta_s, \varphi_s)$ close to the acoustical axis, with subsequent averaging, for statistical improvement of the resulting calibration measurements.

3.2.2 Method 2

An alternative and equivalent method to determine G_0 by echosounder calibration using the metal sphere, may be derived as follows. From Eqs. (11) and (3) one has

$$G_0^2 \cdot \sigma_{bs}^{sph} = \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot \tau_p \cdot [tivs]_R^{sph}}{\lambda_s^2 \cdot F_{VV,s}^2 \cdot \tau_s \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4 \cdot [tivs]_r} \equiv C_1, \quad (24)$$

where C_1 is a constant and known (measured) value for a given calibration measurement. Therefore, by knowing $\sigma_{bs,theory}^{sph}$, the relevant calibration value G_0 is given by $G_0^2 = C_1 / \sigma_{bs,theory}^{sph}$.

Consider a situation where an old (or an arbitrary and erroneous) calibration value is used initially in the calibration measurement, denoted $G_{0,old}$. The corresponding measured value for the backscattering coefficient of the calibration sphere is then $\sigma_{bs,old}^{sph} = C_1 / G_{0,old}^2$. It follows that $C_1 = G_0^2 \cdot \sigma_{bs,theory}^{sph} = G_{0,old}^2 \cdot \sigma_{bs,old}^{sph}$, giving

$$G_0^2 = G_{0,old}^2 \cdot \frac{\sigma_{bs,old}^{sph}}{\sigma_{bs,theory}^{sph}}, \quad (25)$$

corresponding to

$$10\log(G_0) = 10\log(G_{0,old}) + \frac{1}{2} \cdot [TS_{old}^{sph} - TS_{theory}^{sph}], \quad (26)$$

where $TS_{old}^{sph} \equiv 10\log(\sigma_{bs,old}^{sph}/r_1^2)$ is the value measured initially for TS of the calibration sphere when using $G_{0,old}$. Eq. (26) is

on a form that is similar to the equation given in [10] for the EK500 calibration factor “ TS transducer gain”, cf. Eq. (38).

The calibration value G_0 may thus be calculated from the old calibration (or arbitrary and erroneous initial) value, $G_{0,old}$, using Eqs. (24)-(26) and the measured calibration data determining C_1 .

From these equations it follows that Eq. (25) is equivalent to Eq. (22). It thus follows that methods 1 and 2 for determination of G_0 under formulation B, are both generally valid approaches, that lead to the same calibration value for G_0 .

4. “EK500 TYPE” OF POWER BUDGET EQUATIONS, - FORMULATION C

An alternative echo integration formulation of the power budget equations for σ_{bs} and s_v is derived in the following, and denoted “formulation C”. Formulation C appears to correspond to, and generalize, the equations used in the Simrad EK500 echosounder [10] (cf. Section 9.3), and is thus for convenience referred to as an “EK500 type” of power budget equations.

4.1 Echo integration formulation C

The basis for the derivation is the echo integration formulation given by Eqs. (11)-(12) (formulation B). The EK500 manual [10] states that “the TS -measurement is based on the peak value of the echo samples in the sphere echo, whereas the s_A -measurement is based on integration (averaging) of the echo samples”.

First, consider a single scattering target with centre at position (r, θ, φ) . Let $|V_R^{st}|_{\max}$ denote the maximum (“peak voltage”) value of the magnitude of the single target echo, $|V_R^{st}(t)|$, within the time window $[t_{st1}, t_{st2}]$. A slight rearrangement of Eq. (11), by introducing $[tivs]_R^{st} \cdot \tau_p = |V_R^{st}|_{\max}^2 \cdot \tau_p \cdot [tivs]_R^{st} / |V_R^{st}|_{\max}^2$ to account for the particular way of integrating a single target echo indicated above [10], yields

$$\sigma_{bs} = \frac{16\pi^2 \cdot r^4 \cdot e^{4\alpha r} \cdot [tivs]_{R,\max}^{st}}{G_{eff}^2(\theta, \varphi) \cdot \lambda^2 \cdot F_{VV}^2 \cdot [tivs]_r} \quad (27)$$

and

$$\begin{aligned}
TS &= 10\log\left(\frac{[tivs]_{R,\max}^{st}}{[tivs]_r}\right) + 40\log\left(\frac{r}{r_3}\right) + 2\hat{\alpha}r \\
&+ 10\log\left(\frac{16\pi^2 \cdot r_3^4}{\lambda^2 \cdot r_1^2}\right) - 20\log(G_{0,eff}) \\
&- 40\log|\mathbf{B}_i(\theta, \varphi)| - 20\log(F_{VV})
\end{aligned} \quad (28)$$

where the definitions

$$[tivs]_{R,\max}^{st} \equiv |V_R^{st}|_{\max}^2 \cdot \tau_p, \quad (29a)$$

$$G_{\text{eff}}^2(\theta, \varphi) \equiv G^2(\theta, \varphi) \cdot \frac{\tau_{st} \cdot |V_R^{st}|_{\max}^2}{[tivs]_R^{st}}, \quad (29b)$$

and $G_{0,\text{eff}} \equiv G_{\text{eff}}(0,0)$, have been used. $[tivs]_{R,\max}^{st}$ is an erroneous estimate of the single target echo integral $[tivs]_R^{st}$, obtained by using the ‘‘ping duration’’, τ_p , and the constant voltage value, $|V_R^{st}|_{\max}$. $G_{\text{eff}}^2(\theta, \varphi)$ represents an ‘‘effective value’’ of $G^2(\theta, \varphi)$, where a correction factor has been included to compensate for the use of $[tivs]_{R,\max}^{st}$ in place of $[tivs]_R^{st}$, in σ_{bs} . Let A^2 denote the reciprocal of this (non-dimensional) correction factor, so that from Eq. (29),

$$G^2(\theta, \varphi) = G_{\text{eff}}^2(\theta, \varphi) \cdot A^2, \quad (30a)$$

$$A^2 \equiv \frac{[tivs]_R^{st}}{\tau_{st} \cdot |V_R^{st}|_{\max}^2} = \frac{\tau_p}{\tau_{st}} \cdot \frac{[tivs]_R^{st}}{[tivs]_{R,\max}^{st}}. \quad (30b)$$

From Eqs. (30) and (3)-(4) it follows that

$$G_{\text{eff}}^2(\theta, \varphi) = G^2(\theta, \varphi) \cdot A^{-2} = G_0^2 \cdot |\mathbf{B}_i(\theta, \varphi)|^4 \cdot A^{-2} \\ = G_{0,\text{eff}}^2 \cdot |\mathbf{B}_i(\theta, \varphi)|^4, \quad (31)$$

where

$$G_{0,\text{eff}}^2 \equiv G_{\text{eff}}^2(0,0) = G_0^2 \cdot A^{-2}. \quad (32)$$

Next, consider volume backscattering from V_p . The expressions for s_v and S_v used in survey operation are then given by Eqs. (12) and (21), respectively (i.e., the same expressions as for formulation B).

Eqs. (27) and (12), or alternatively Eqs. (28) and (21), thus constitute formulation C of the power budget equations for σ_{bs} and s_v , on ‘‘normal’’ and logarithmic (dB) forms, respectively.

From these equations, formulation C is seen to involve two calibration factors, $G_{0,\text{eff}}$ and G_0 . The EK500 manual [10] refers to two calibration measurements, ‘‘TS calibration’’ and ‘‘ s_A calibration’’, involving the two calibration factors ‘‘TS transducer gain’’ (related to σ_{bs}) and ‘‘ S_v transducer gain’’ (related to s_v), respectively.

In the following, expressions are derived for the calibration factors involved in formulation C. Two approaches are used. The first approach (denoted ‘‘method 1’’) gives the two formulation C calibration factors $G_{0,\text{eff}}$ and G_0 in terms of the echosounder parameters that are measured at calibration. The second approach (denoted ‘‘method 2’’) is used to (a) establish alternative relationships for use in calibration, and (b) establish the relationships between $G_{0,\text{eff}}$ and G_0 , and the two cali-

bration factors used in the EK500 manual [10], ‘‘TS transducer gain’’ and ‘‘ S_v transducer gain’’.

4.2 Formulation C calibration factor $G_{0,\text{eff}}$ (‘‘TS transducer gain’’)

4.2.1 Method 1

Consider a calibration situation, with the centre of the calibration sphere located at position $(r_s, \theta_s, \varphi_s)$. From Eqs. (27) and (31), $G_{0,\text{eff}}$ can be determined from calibration measurement data as

$$G_{0,\text{eff}}^2 \equiv \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot [tivs]_{R,\max}^{sph}}{\lambda_s^2 \cdot \sigma_{bs,\text{theory}}^{sph} \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4 \cdot F_{VV,s}^2 \cdot [tivs]_T}. \quad (33)$$

The corresponding logarithmic expression becomes

$$20\log(G_{0,\text{eff}}) = 10\log\left(\frac{[tivs]_{R,\max}^{sph}}{[tivs]_T}\right) + 40\log\left(\frac{r_s}{r_3}\right) \\ + 2\hat{\alpha}r_s + 10\log\left(\frac{16\pi^2 \cdot r_s^4}{\lambda_s^2 \cdot r_1^2}\right) - TS_{\text{theory}}^{sph} \\ - 40\log|\mathbf{B}_i(\theta_s, \varphi_s)| - 20\log(F_{VV,s}) \quad (34)$$

Here, the following relationships have been used,

$$[tivs]_{R,\max}^{sph} \equiv |V_R^{sph}|_{\max}^2 \cdot \tau_p, \\ A^2 \equiv \frac{[tivs]_R^{sph}}{\tau_s \cdot |V_R^{sph}|_{\max}^2} = \frac{\tau_p}{\tau_s} \cdot \frac{[tivs]_R^{sph}}{[tivs]_{R,\max}^{sph}}, \quad (35)$$

where $[tivs]_{R,\max}^{sph}$ is an erroneous estimate of the echo integral for the calibration sphere, $[tivs]_R^{sph}$, and $|V_R^{sph}|_{\max}$ denotes the maximum (‘‘peak voltage’’) value of the magnitude of the calibration sphere echo, $|V_R^{sph}(t)|$.

4.2.2 Method 2

An alternative and equivalent method for determining $G_{0,\text{eff}}$ in echosounder calibration using the metal sphere, is derived in the following. It is likely that a related method may have been used to obtain the result postulated for the ‘‘TS transducer gain’’ in the EK500 manual [10], cf. Eq. (38).

From Eqs. (27) and (31) one has

$$G_{0,\text{eff}}^2 \cdot \sigma_{bs}^{sph} = \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot [tivs]_{R,\max}^{sph}}{\lambda_s^2 \cdot F_{VV,s}^2 \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4 \cdot [tivs]_T} \equiv C_2, \quad (36)$$

where C_2 is a constant and known (measured) value for a given calibration measurement. Therefore, by knowing $\sigma_{bs,theory}^{sph}$, the calibration value $G_{0,eff}$ is given from $G_{0,eff}^2 = C_2 / \sigma_{bs,theory}^{sph}$.

Consider a situation where an old (or an arbitrary and erroneous) calibration value is used initially in the calibration measurement, denoted $G_{0,eff,old}$. The corresponding measured value for the backscattering coefficient of the calibration sphere is $\sigma_{bs,old}^{sph} = C_2 / G_{0,eff,old}^2$. It follows that $C_2 = G_{0,eff}^2 \cdot \sigma_{bs,theory}^{sph} = G_{0,eff,old}^2 \cdot \sigma_{bs,old}^{sph}$, giving

$$\boxed{G_{0,eff}^2 = G_{0,eff,old}^2 \cdot \frac{\sigma_{bs,old}^{sph}}{\sigma_{bs,theory}^{sph}}}, \quad (37)$$

corresponding to

$$\boxed{10\log(G_{0,eff}) = 10\log(G_{0,eff,old}) + \frac{1}{2} \cdot [TS_{eff,old}^{sph} - TS_{theory}^{sph}]}, \quad (38)$$

where $TS_{eff,old}^{sph} \equiv 10\log(\sigma_{bs,old}^{sph} / r_1^2)$ is the value measured initially for TS of the calibration sphere by using $G_{0,eff,old}$. By defining TS transducer gain $\equiv 10\log(G_{0,eff})$, Eq. (38) becomes identical to the expression postulated for the EK500 calibration factor “ TS transducer gain” in the EK500 manual [10].

In this “method 2”, the calibration value $G_{0,eff}$ is thus determined from the old calibration (or arbitrary and erroneous initial) value, $G_{0,eff,old}$, using Eqs. (36)-(37) and the measured calibration data determining C_2 .

Using these equations, it follows that Eqs. (37)-(38) are equivalent to Eqs. (33)-(34). Consequently, methods 1 and 2 for determination of $G_{0,eff}$ under formulation C, are both valid approaches, that lead to the same calibration value for $G_{0,eff}$.

4.3 Formulation C calibration factor G_0 (“ S_v transducer gain”)

4.3.1 Method 1

From Eqs. (33), (32) and (30), it follows that G_0 , and its corresponding logarithmic quantity $10\log(G_0)$, are given by the same expressions as for formulation B, Eqs. (22) and (23), respectively. Measurement quantities involved to determine the calibration factors $G_{0,eff}$ and G_0 in echosounder calibration are the same as for formulation B, in addition to $|V_R^{sph}|_{\max}$.

4.3.2 Method 2

An alternative and equivalent approach for determining G_0 in echosounder calibration using the calibration sphere, is derived in the following. This method is analogous to the approach used in Section 3.2.2 for formulation B, leading to Eqs. (24)-

(25). It is likely that a related method may have been used to obtain the result postulated in the EK500 manual [10] for the calibration factor “ S_v transducer gain”, cf. Eq. (45).

Let s_v^{sph} denote the volume backscattering coefficient of the calibration sphere with centre at position $(r_s, \theta_s, \varphi_s)$. From Eqs. (27) and (12) (formulation C), by forming the ratio $s_v^{sph} / \sigma_{bs}^{sph}$, and using Eqs. (31), (32) and (35), one finds that

$$s_v^{sph} = \frac{2}{c_{0s} \tau_p} \cdot \frac{|B_i(\theta_s, \varphi_s)|^4}{\psi \cdot r_s^2} \cdot \sigma_{bs}^{sph}. \quad (39)$$

The corresponding area backscattering coefficient [15] of the calibration sphere is from Eq. (39) given as

$$\begin{aligned} s_a^{sph} &= \int_{r_{s1}}^{r_{s2}} s_v^{sph} dr \approx s_v^{sph} (r_{s2} - r_{s1}) \\ &= s_v^{sph} \cdot \frac{1}{2} c_{0s} \tau_s = \frac{\tau_s}{\tau_p} \cdot \frac{|B_i(\theta_s, \varphi_s)|^4}{\psi \cdot r_s^2} \cdot \sigma_{bs}^{sph}, \end{aligned} \quad (40)$$

where $r_{s1} = \frac{1}{2} c_0 t_{s1}$ and $r_{s2} = \frac{1}{2} c_0 t_{s2}$ are the ranges corresponding to the arrival times t_{s1} and t_{s2} of the start and end of the sphere echo, $V_R^{sph}(t)$, respectively. It follows from Eq. (40) that

$$\boxed{s_{a,theory}^{sph} = \frac{\tau_s}{\tau_p} \cdot \frac{|B_i(\theta_s, \varphi_s)|^4}{\psi \cdot r_s^2} \cdot \sigma_{bs,theory}^{sph}} \quad (41)$$

is a known value of the area backscattering coefficient of the sphere, given from calibration measurements and the known (calculated) value $\sigma_{bs,theory}^{sph}$.⁶ Now, from Eq. (12) one also has

$$\begin{aligned} s_a^{sph} &= \int_{r_{s1}}^{r_{s2}} s_v^{sph} dr \approx s_v^{sph} (r_{s2} - r_{s1}) \\ &= s_v^{sph} \cdot \frac{1}{2} c_{0s} \tau_s = \frac{16\pi^2 \cdot r_s^2 \cdot e^{4\alpha r_s} \cdot [tivs]_R^{sph}}{G_0^2 \cdot \psi \cdot \lambda_s^2 \cdot F_{VV,s}^2 \cdot [tivs]_F}, \end{aligned} \quad (42)$$

where $[tivs]_R^{gv}$ and the gate opening time τ_g in Eq. (12) have been set to $[tivs]_R^{sph}$ and τ_s , respectively, since at calibration the time integration is performed over the sphere echo. From Eq. (42) it follows that

$$G_0^2 \cdot s_a^{sph} = \frac{16\pi^2 \cdot r_s^2 \cdot e^{4\alpha r_s} \cdot [tivs]_R^{sph}}{\psi \cdot \lambda_s^2 \cdot F_{VV,s}^2 \cdot [tivs]_F} \equiv C_3, \quad (43)$$

⁶ A similar expression as Eq. (41) is given by [10], but without $|B_i(\theta_s, \varphi_s)|$ and the factor τ_s / τ_p . That expression thus relates to calibration sphere located on the acoustic axis, $\theta_s = \varphi_s = 0$, and $\tau_s = \tau_p$.

where C_3 is a constant and known (measured) value for a given calibration measurement. Thus, by knowing $\sigma_{bs,theory}^{sph}$, and thus $s_{a,theory}^{sph}$, the calibration value G_0 is given by $G_0^2 = C_3 / s_{a,theory}^{sph}$.

Consider a situation where an old (or an arbitrary and erroneous) calibration value is used initially in the calibration measurement, denoted $G_{0,old}$. The corresponding measured value for the area backscattering coefficient of the sphere is then $s_{a,old}^{sph} = C_3 / G_{0,old}^2$. It follows that $C_3 = G_0^2 \cdot s_{a,theory}^{sph} = G_{0,old}^2 \cdot s_{a,old}^{sph}$, giving

$$\boxed{G_0^2 = G_{0,old}^2 \cdot \frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}}}, \quad (44)$$

corresponding to

$$\boxed{10\log(G_0) = 10\log(G_{0,old}) + \frac{1}{2} \cdot 10\log\left(\frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}}\right)}. \quad (45)$$

By defining S_v transducer gain $\equiv 10\log(G_0)$, Eq. (45) becomes identical to the expression postulated for the EK500 calibration factor “ S_v transducer gain” in the EK500 manual [10].

In this approach, G_0 is determined from the old calibration (or arbitrary and erroneous initial) value, $G_{0,old}$, using Eqs. (41), (43)-(44), and the measured calibration data determining C_3 .

From these equations it is readily shown that Eqs. (44)-(45) are equivalent to Eqs. (22)-(23). It thus turns out that methods 1 and 2 for determination of G_0 under formulation C, are both valid approaches, and lead to the same calibration value for G_0 .

5. “EK60 TYPE” OF POWER BUDGET EQUATIONS, - FORMULATION D

A second alternative echo integration formulation of the power budget equations for σ_{bs} and s_v is derived in the following, and denoted “formulation D”. Formulation D appears to correspond to and generalize the equations used for the Simrad EK60, ES60, ME70 and MS70 echosounder and sonar systems, given by Ona *et al.* [13] (cf. Section 9.4). For convenience, this formulation is thus referred to as an “EK60 type” of power budget equations.

The literature on the mentioned echosounders [16-19] refers to calibration measurements involving two calibration factors, denoted “Gain” and “ s_a correction”. It is stated [17] that the “ s_a correction value represents the correction required to the S_v constant to harmonize the TS and s_A measurements”.

5.1 Echo integration formulation D

The basis for the derivation is the echo integration formulation given by Eqs. (11)-(12) (formulation B). First, consider a single scattering target with centre at position (r, θ, φ) . As for EK500, it is assumed that the statement [10] “the TS-measurement is based on the peak value of the echo samples in the sphere echo”, is valid also for the more recent generation of Simrad echosounder and sonar systems, EK60, etc. [16-19].

For the single target, the analysis is then identical to the one leading to Eqs. (27)-(32). It follows that in formulation D, σ_{bs} and the corresponding TS are given by the same expressions as in formulation C, Eqs. (27) and (28), respectively.

Next, consider volume backscattering from V_p , given by Eqs. (11) and (32). In formulation D, the parameter “A”, defined by Eqs. (30) and (35), is given a key role, replacing G_0 as a calibration factor. Thus, to harmonize terminology with [13,16-19], define

$$s_{a,corr} \equiv A, \quad (46)$$

so that, from Eqs. (30), (35), (16), and (17), it follows that

$$s_{a,corr}^2 \equiv \frac{[tivs]_R^{st}}{\tau_{st} \cdot |V_R^{st}|_{\max}^2} = \frac{\tau_p}{\tau_{st}} \cdot \frac{[tivs]_R^{st}}{[tivs]_{R,\max}^{st}} = \frac{\Pi_R^{st}}{\Pi_{R,\max}^{st}}, \quad (47)$$

$$s_{a,corr}^2 \equiv \frac{[tivs]_R^{sph}}{\tau_s \cdot |V_R^{sph}|_{\max}^2} = \frac{\tau_p}{\tau_s} \cdot \frac{[tivs]_R^{sph}}{[tivs]_{R,\max}^{sph}} = \frac{\Pi_R^{sph}}{\Pi_{R,\max}^{sph}}, \quad (48)$$

for a single target (e.g. individual fish) and the calibration sphere, respectively. Here, the relations $\Pi_{R,\max}^{st} = (R_E / |Z_E|^2) \cdot |V_R^{st}|_{\max}^2$ and $\Pi_{R,\max}^{sph} = (R_E / |Z_E|^2) \cdot |V_R^{sph}|_{\max}^2$ have been used, which follow from Eqs. (16)-(17). $s_{a,corr}^2$ thus represents the (non-dimensional) ratio of the actual electrical power in the single-target (or calibration sphere) echo, to the “maximum electrical power” of that echo (i.e., the power obtained by using τ_p and the constant voltage $|V_R^{st}|_{\max}$ (or $|V_R^{sph}|_{\max}$) in echo integration, instead of τ_{st} (or τ_s) and the voltage signal $V_R^{st}(t)$ (or $V_R^{sph}(t)$). From Eqs. (31)-(32) it follows that

$$G_{\text{eff}}^2(\theta, \varphi) = G^2(\theta, \varphi) \cdot s_{a,corr}^{-2} = G_0^2 \cdot |\mathbf{B}_i(\theta, \varphi)|^4 \cdot s_{a,corr}^{-2}, \quad (49)$$

$$= G_{0,\text{eff}}^2 \cdot |\mathbf{B}_i(\theta, \varphi)|^4$$

$$G_{0,\text{eff}}^2 = G_0^2 \cdot s_{a,corr}^{-2}. \quad (50)$$

In formulation D, thus, the calibration factor G_0 is replaced by $G_{0,\text{eff}}$ and $s_{a,corr}$, so that in this formulation, the two calibration factors are $G_{0,\text{eff}}$ and $s_{a,corr}$. Eqs. (11) and (50) lead to

$$s_v = \frac{32\pi^2 \cdot r^2 \cdot e^{4\alpha r} \cdot [tivs]_R^{gv}}{G_{0,eff}^2 \cdot \psi \cdot \lambda^2 c_0 \cdot \tau_g \cdot s_{a,corr}^2 \cdot F_{VV}^2 \cdot [tivs]_T}, \quad (51)$$

corresponding to

$$S_v = 10\log\left(\frac{[tivs]_R^{gv}}{[tivs]_T}\right) + 20\log\left(\frac{r}{r_3}\right) + 2\hat{\alpha}r + 10\log\left(\frac{32\pi^2 \cdot r_2 \cdot r_3^2}{\lambda^2 c_0 \cdot \tau_g}\right) - 20\log(G_{0,eff}) - 10\log\psi - 2S_{a,corr} - 20\log(F_{VV}), \quad (52)$$

where $S_{a,corr} \equiv 10\log(s_{a,corr})$ is the “ S_a correction” [16-19].

Eqs. (27) and (51), or alternatively, Eqs. (28) and (52), constitute formulation D of the power budget equations for σ_{bs} and s_v , on “normal” and logarithmic (dB) forms, respectively.

From these equations, formulation D is seen to involve two calibration factors, $G_{0,eff}$ and $s_{a,corr}$. In the following, expressions are derived for these. Two approaches are used. The first approach (denoted “method 1”) gives the two formulation D calibration factors $G_{0,eff}$ and $s_{a,corr}$ in terms of the echosounder parameters that are measured at calibration. The second approach (denoted “method 2”) is used to (a) establish alternative relationships for use in calibration, and (b) establish relationships between the two formulation D calibration factors, $G_{0,eff}$ and $s_{a,corr}$, and the two calibration factors used for the EK60 etc- systems, “Gain” and “ S_a correction” [13,16-19].

5.2 Formulation D calibration factor $G_{0,eff}$ (“Gain”)

Consider a calibration situation, with the centre of the calibration sphere located at position $(r_s, \theta_s, \varphi_s)$. Since the “effective” axial transducer gain $G_{0,eff}$ is used as a calibration factor in both of formulations C and D, it follows that for formulation D, $G_{0,eff}$ and $10\log(G_{0,eff})$ are given by the same expressions as for formulation C, Eqs. (33) and (34), respectively (“method 1”), or equivalently, Eqs. (37) and (38), respectively (“method 2”).

By defining $Gain \equiv 10\log_{10}(G_{0,eff})$, Eq. (38) thus appear to represent a similar expression for EK60 and related echosounder/sonar systems [16-19], as TS transducer gain $\equiv 10\log(G_{0,eff})$ does for EK500 [10].

5.3 Formulation D calibration factor $s_{a,corr}$ (“ S_a correction”)

5.3.1 Method 1

From Eqs. (22) and (50) it follows that $s_{a,corr}$ can be determined from calibration measurement data as

$$s_{a,corr}^2 = \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot \tau_p \cdot [tivs]_R^{sph}}{G_{0,eff}^2 \cdot \lambda_s^2 \cdot \tau_s \cdot \sigma_{bs,theory}^{sph} \cdot |B_i(\theta_s, \varphi_s)|^4 \cdot F_{VV,s}^2 \cdot [tivs]_T}, \quad (53)$$

giving

$$2S_{a,corr} = 10\log\left(\frac{[tivs]_R^{sph}}{[tivs]_T}\right) + 40\log\left(\frac{r_s}{r_3}\right) + 2\hat{\alpha}r_s + 10\log\left(\frac{\tau_p}{\tau_s}\right) + 10\log\left(\frac{16\pi^2 \cdot r_s^4}{\lambda_s^2 \cdot r_1^2}\right) - TS_{theory}^{sph} - 20\log(G_{0,eff}) - 40\log|B_i(\theta_s, \varphi_s)| - 20\log(F_{VV,s}), \quad (54)$$

Measurement quantities involved to determine the calibration factors $G_{0,eff}$ and $s_{a,corr}$ in echosounder calibration are the same as for formulation C.

5.3.2 Method 2

An alternative and equivalent method for determination of $s_{a,corr}$ in echosounder calibration using the metal calibration sphere, is derived in the following. This method is analogous to the approach described in Section 4.3.2 for formulation C, leading to Eqs. (44)-(45).

From Eqs. (27) and (51) (formulation D), by forming the ratio $s_v^{sph}/\sigma_{bs}^{sph}$, and using Eqs. (48)-(49), one obtains Eqs. (39)-(40). As for formulation C, $s_{a,theory}^{sph}$ is then known and given by Eq. (41). Now, from Eq. (51) one has, using the same approach with respect to $[tivs]_R^{gv}$ and τ_g as for Eq. (42),

$$s_a^{sph} = \int_{r_{s1}}^{r_{s2}} s_v^{sph} dr \approx s_v^{sph}(r_{s2} - r_{s1}) = s_v^{sph} \cdot \frac{1}{2} c_0 \tau_s = \frac{16\pi^2 \cdot r_s^2 \cdot e^{4\alpha r_s} \cdot [tivs]_R^{sph}}{G_{0,eff}^2 \cdot s_{a,corr}^2 \cdot \psi \cdot \lambda_s^2 \cdot F_{VV,s}^2 \cdot [tivs]_T}, \quad (55)$$

From Eq. (55) it follows that

$$s_{a,corr}^2 \cdot s_a^{sph} = \frac{16\pi^2 \cdot r_s^2 \cdot e^{4\alpha r_s} \cdot [tivs]_R^{sph}}{G_{0,eff}^2 \cdot \psi \cdot \lambda_s^2 \cdot F_{VV,s}^2 \cdot [tivs]_T} \equiv C_4, \quad (56)$$

where C_4 is a constant and known (measured) value for a given calibration measurement. Consequently, by knowing $\sigma_{bs,theory}^{sph}$, and thus $s_{a,theory}^{sph}$ from Eq. (41), the relevant calibration value $s_{a,corr}$ is given from $s_{a,corr}^2 = C_4 / s_{a,theory}^{sph}$.

Similar to the approach used in Section 4.3.2, consider a situation where an old (or an arbitrary and erroneous) calibration value is used initially in the calibration measurement, denoted $s_{a,corr,old}$. The corresponding measured value for the area backscattering coefficient of the sphere is then

$s_{a,old}^{sph} = C_4 / s_{a,corr,old}^2$. It follows that $C_4 = s_{a,corr}^2 \cdot s_{a,theory}^{sph} = s_{a,corr,old}^2 \cdot s_{a,old}^{sph}$, giving

$$s_{a,corr}^2 = s_{a,corr,old}^2 \cdot \frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}}. \quad (57)$$

By defining $S_{a,corr,old} \equiv 10\log(s_{a,corr,old})$, the corresponding logarithmic expression becomes

$$S_{a,corr} = S_{a,corr,old} + \frac{1}{2} \cdot 10\log\left(\frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}}\right). \quad (58)$$

In this method, $s_{a,corr}$ is determined from the old calibration (or arbitrary and erroneous initial) value, $s_{a,corr,old}$, using Eqs. (41), (56)-(57), and the measured calibration data determining C_4 .

From these equations it is readily shown that Eqs. (57)-(58) are equivalent to Eqs. (53)-(54). Consequently, it is shown that methods 1 and 2 for determination of $s_{a,corr}$ under formulation D, are generally valid approaches, that lead to the same calibration result for $s_{a,corr}$.

5.3.3 Method 3

Another alternative and equivalent method for determination of $s_{a,corr}$ in calibration using the metal calibration sphere, may also be of interest. From Eq. (50) one has

$$s_{a,corr} = \frac{G_0}{G_{0,eff}}, \quad (59a)$$

so that

$$S_{a,corr} = 10\log(G_0) - 10\log(G_{0,eff}). \quad (59b)$$

To determine $s_{a,corr}$ using Eqs. (59), the EK60 type of echosounder may be operated more as an EK500, where $G_0 = G_{0,eff} \cdot s_{a,corr}$ is treated as one single parameter, and determined in echosounder calibration [s_v measurement, using Eq. (45)], together with $G_{0,eff}$ [TS measurement, using Eq. (38)].

It does not appear as any such expressions as Eqs. (38), (58) and (59b) (nor equivalent expressions) have been given in the available literature on the EK60 and related systems [13,16-19]. The author has however been informed [20] that Eqs. (38) and (59b) are implemented and used in the EK60 system.⁷

⁷ In use of Eq. (59b), the following notation is employed [20]: $G_{SV} \equiv 10\log(G_0)$ and $G_{TS} \equiv 10\log(G_{0,eff})$, so that Eq. (59b) becomes $S_{a,corr} = G_{SV} - G_{TS}$

6. ALTERNATIVE POWER BUDGET EQUATIONS, - FORMULATION E

A third alternative echo integration formulation of the power budget equations for σ_{bs} and s_v (denoted ‘‘formulation E’’) is derived in the following, in terms of two calibration factors, $G_{0,eff}$ and τ_{eff} .

The background for addressing this formulation is the following. Ona *et al.* [13] stated - in terms of the terminology used here, and slightly incorrectly, since they combined logarithmic and linear quantities in the same equation, cf. Eqs. (60)-(61) below - that ‘‘the sum of τ_p and $S_{a,corr}$ equals the effective pulse duration’’^{8,9}. Although $A \equiv s_{a,corr}$ appears to originate from, and compensate for, a specific signal processing solution used in EK500 [10] and EK60 [13,16], with respect to echo integration of single target echoes, as explained in Sections 4.1 and 5.1, $s_{a,corr}$ may alternatively (in analogy with ref. [13]) be interpreted as a correction factor to the integration time for the gated volume, τ_g , as shown below.

As such a formulation has been discussed for possible use in future echosounders and sonars [20], the set of equations and calibration factors for such a formulation is derived in the following. The validity of interpreting τ_{eff} as an ‘‘effective pulse duration’’ [13] is also explored.

6.1 Echo integration formulation E

The basis for the derivation is the echo integration formulation given by Eqs. (27) and (51) (Formulation D). From Eq. (51), define

$$\tau_{eff} \equiv \tau_g \cdot s_{a,corr}^2, \quad (60)$$

which on logarithmic form becomes similar to an expression used by [13] (in their expression for S_v),

$$10\log(\tau_{eff} / \tau_{ref}) = 10\log(\tau_g / \tau_{ref}) + 2S_{a,corr}. \quad (61)$$

τ_{ref} is here a reference time interval for τ_{eff} (chosen equal to 1 s). It follows from Eqs. (60) and (50) that

$$\tau_{eff} = \tau_g \cdot \frac{G_0^2}{G_{0,eff}^2}, \quad (62)$$

and, from Eqs. (60) and (47)-(48), that

$$\tau_{eff} = \tau_g \cdot \frac{\prod_R^{st}}{\prod_{R,max}^{st}} = \tau_g \cdot \frac{\prod_R^{sph}}{\prod_{R,max}^{sph}}. \quad (63)$$

⁸ Note that in [13], it is likely that $\tau_g = \tau_p$ has been used, cf. the discussion in Section 3.1.

⁹ An alternative statement may be proposed: ‘‘the product of τ_g and $s_{a,corr}^2$ equals the effective echo integration time’’, cf. Eq. (60) and Section 9.5.

Consequently, τ_{eff} (s) is the gate opening time (as used for volume backscattering in oceanic surveying), scaled by the ratio of the actual electrical power to the “maximum electrical power” of a single target echo (e.g. a fish, or a calibration sphere). τ_{eff} may thus be interpreted as an “effective echo integration time”. The interpretation of τ_{eff} as an “effective pulse duration”, as used by [13], may be misleading, cf. Section 9.5.

In formulation E, $s_{a,corr}$ (used in formulation D), is replaced by τ_{eff} in the expression for s_v , so that the two calibration factors of this formulation are $G_{0,eff}$ and τ_{eff} . Eqs. (50), (60), and (62) give the relationships between the various calibration factors used in formulations B-E.

Since $G_{0,eff}$ is used in all of formulations C-E, the single-target backscattering coefficient σ_{bs} and the corresponding target strength TS , are in formulation E given by the same expressions as in formulations C and D, Eqs. (27) and (28), respectively.

For the volume backscattering coefficient s_v , Eqs. (51) and (60) lead to

$$s_v = \frac{32\pi^2 \cdot r^2 \cdot e^{4\alpha r} \cdot [tivs]_R^{sv}}{G_{0,eff}^2 \cdot \psi \cdot \lambda^2 c_0 \cdot \tau_{eff} \cdot F_{VV}^2 \cdot [tivs]_R} \quad (64)$$

The corresponding logarithmic expression, giving the volume backscattering strength, is

$$S_v = 10\log\left(\frac{[tivs]_R^{sv}}{[tivs]_R}\right) + 20\log\left(\frac{r}{r_3}\right) + 2\hat{\alpha}r + 10\log\left(\frac{32\pi^2 \cdot r^2 \cdot r_3^2}{\lambda^2 c_0 \cdot \tau_{eff}}\right) - 20\log(G_{0,eff}) - 10\log\psi - 20\log(F_{VV}) \quad (65)$$

Eqs. (27) and (64), or alternatively, Eqs. (28) and (65), thus constitute formulation E of the power budget equations for σ_{bs} and s_v , on “normal” and logarithmic (dB) forms, respectively.

In the following, expressions are derived for the calibration factors involved in formulation E. Two approaches are used. The first approach (denoted “method 1”) gives the two formulation E calibration factors $G_{0,eff}$ and τ_{eff} in terms of the echosounder parameters that are measured at calibration. The second approach (denoted “method 2”) is used to (a) establish alternative relationships for use in calibration, and (b) express $G_{0,eff}$ and τ_{eff} on forms similar to those used for the formulation C and D calibration factors in Sections 4 and 5.

6.2 Formulation E calibration factor $G_{0,eff}$ (“Gain”)

Consider a calibration situation, with the centre of the calibration sphere located at position $(r_s, \theta_s, \varphi_s)$. Since the “effective”

axial transducer gain $G_{0,eff}$ is used as a calibration factor in all three formulations C, D and E, it follows that in formulation E, $G_{0,eff}$ and $10\log(G_{0,eff})$ are given by the same expressions as for formulations C and D, i.e., Eqs. (33) and (34), respectively (“method 1”), or equivalently, Eqs. (37) and (38), respectively (“method 2”).

Similar to in Section 5.2, one has $Gain \equiv 10\log(G_{0,eff})$.

6.3 Formulation E calibration factor τ_{eff} (“Effective echo integration time”)

6.3.1 Method 1

It follows from Eqs. (53) and (60) that τ_{eff} can be determined from calibration measurement data as

$$\tau_{eff} = \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot \tau_p \cdot [tivs]_R^{sph}}{G_{0,eff}^2 \cdot \lambda_s^2 \cdot F_{VV,s}^2 \cdot [tivs]_R \cdot \sigma_{bs,\theta,\varphi}^{sph} \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4} \quad (66)$$

where $\tau_g = \tau_s$ has been used since the time integration is performed over the sphere echo. The corresponding logarithmic expression is

$$10\log\left(\frac{\tau_{eff}}{\tau_{ref}}\right) = 10\log\left(\frac{[tivs]_R^{sph}}{[tivs]_R}\right) + 40\log\left(\frac{r_s}{r_3}\right) + 2\hat{\alpha}r_s + 10\log\left(\frac{\tau_p}{\tau_{ref}}\right) + 10\log\left(\frac{16\pi^2 \cdot r_s^4}{\lambda_s^2 \cdot r_1^2}\right) - 20\log(F_{VV,s}) - 20\log(G_{0,eff}) - 40\log|\mathbf{B}_i(\theta_s, \varphi_s)| - TS_{\theta,\varphi}^{sph} \quad (67)$$

Quantities to be measured in echosounder calibration for determination of $G_{0,eff}$ and τ_{eff} are the same as for formulations C and D.

6.3.2 Method 2

An alternative and equivalent method for determination of τ_{eff} in echosounder calibration using the metal sphere, is derived in the following.

From Eq. (60) one has $\tau_{eff} \equiv \tau_s \cdot s_{a,corr}^2$ for the calibration situation, where $\tau_g = \tau_s$ has been used since the time integration is performed over the sphere echo. Insertion in Eq. (56) leads to

$$\tau_{eff} \cdot s_a^{sph} = C_5 \quad (68)$$

where $C_5 \equiv C_4 \cdot \tau_s$ is a constant and known (measured) value for a given calibration measurement. Consequently, by knowing $\sigma_{bs,\theta,\varphi}^{sph}$, and thus $s_{a,\theta,\varphi}^{sph}$ from Eq. (41), the relevant calibration value τ_{eff} is known and given from $\tau_{eff} = C_5 / s_{a,\theta,\varphi}^{sph}$.

Similar to the approaches used in Sections 4.3.2 and 5.3.2, consider a situation where an old (or an arbitrary and

erroneous) calibration value is used initially in the calibration measurement, denoted $\tau_{eff,old}$. From Eq. (68) the corresponding measured value for the area backscattering coefficient of the sphere is $s_{a,old}^{sph} = C_s / \tau_{eff,old}$. It follows that $C_s = \tau_{eff} \cdot s_{a,theory}^{sph} = \tau_{eff,old} \cdot s_{a,old}^{sph}$, giving

$$\tau_{eff} = \tau_{eff,old} \cdot \frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}}. \quad (69)$$

The corresponding logarithmic expression becomes

$$10 \log \left(\frac{\tau_{eff}}{\tau_{ref}} \right) = 10 \log \left(\frac{\tau_{eff,old}}{\tau_{ref}} \right) + 10 \log \left(\frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}} \right). \quad (70)$$

In this approach, τ_{eff} is thus determined from an old calibration (or an arbitrary and erroneous initial) value, $\tau_{eff,old}$, using Eqs. (41), (68)-(69), and the measured calibration data determining C_s .

From these equations it is readily shown that Eqs. (69)-(70) are equivalent to Eqs. (66)-(67). It follows that method 1 and 2 for determination of τ_{eff} under formulation E, are both generally valid approaches, that lead to the same calibration result for τ_{eff} .

6.3.3 Method 3

Another alternative and equivalent method for determination of τ_{eff} in calibration using the metal calibration sphere, may also be of interest. From Eq. (62) one has

$$10 \log \left(\frac{\tau_{eff}}{\tau_{ref}} \right) = 10 \log \left(\frac{\tau_g}{\tau_{ref}} \right) + 2 [10 \log(G_0) - 10 \log(G_{0,eff})]. \quad (71)$$

To determine τ_{eff} using Eq. (71), the EK60 type of echosounder may be operated more as an EK500, where $G_0^2 = G_{0,eff}^2 \cdot (\tau_{eff} / \tau_g)$ is treated as one single parameter and determined in echosounder calibration [s_v measurement, using Eq. (45)], together with $G_{0,eff}$ [TS meas., using Eq. (38)].

7. A GENERIC AND UNIFYING ECHO INTEGRATION FUNCTIONAL RELATIONSHIP

Eqs. (11)-(12), together with Eq. (22) for the calibration factor G_0 , constitute formulation B of the functional relationship for fish abundance estimation. Similarly, Eqs. (27) and (12), together with Eqs. (33) and (22) for the calibration factors $G_{0,eff}$ and G_0 , constitute formulation C. Thirdly, Eqs. (27) and (51), together with Eqs. (33) and (53) for the calibration factors

$G_{0,eff}$ and $s_{a,corr}$, constitute formulation D of the functional relationship for fish abundance estimation. Fourthly, Eqs. (27) and (64), together with Eqs. (33) and (66) for the calibration factors $G_{0,eff}$ and τ_{eff} , constitute formulation E.

Now, for formulation B, insertion of Eq. (22) into Eqs. (11)-(12), yields

$$\sigma_{bs} = e^{4\alpha(r-r_s)} \cdot \left(\frac{r}{r_s} \right)^4 \cdot \left(\frac{\lambda_s}{\lambda} \right)^2 \cdot \left(\frac{F_{VV,s}}{F_{VV}} \right)^2 \cdot \frac{\tau_s}{\tau_{st}} \cdot \frac{[tivs]_R^{st}}{[tivs]_R^{sph}} \cdot \left| \frac{\mathbf{B}_i(\theta_s, \varphi_s)}{\mathbf{B}_i(\theta, \varphi)} \right|^4 \cdot \sigma_{bs,theory}^{sph}, \quad (72)$$

$$s_v = e^{4\alpha(r-r_s)} \cdot \frac{r^2}{r_s^4} \cdot \left(\frac{\lambda_s}{\lambda} \right)^2 \cdot \left(\frac{F_{VV,s}}{F_{VV}} \right)^2 \cdot \frac{2}{c_0 \tau_g \tau_p} \cdot \frac{\tau_s}{\tau_p} \cdot \frac{[tivs]_R^{st}}{[tivs]_R^{sph}} \cdot \frac{|\mathbf{B}_i(\theta_s, \varphi_s)|^4}{\psi} \cdot \sigma_{bs,theory}^{sph}, \quad (73)$$

respectively, where $[tivs]_r$ has been assumed invariant from calibration to oceanic survey operation (i.e., the same transmit pulse $V_T(t)$ and integration time τ_p are used). The equivalent logarithmic expressions are

$$TS = 40 \log \left(\frac{r}{r_s} \right) + 2\hat{\alpha}(r-r_s) + 20 \log \left(\frac{\lambda_s}{\lambda} \right) + 20 \log \left(\frac{F_{VV,s}}{F_{VV}} \right) + 10 \log \left(\frac{\tau_s}{\tau_{st}} \right) + 10 \log \left(\frac{[tivs]_R^{st}}{[tivs]_R^{sph}} \right) + 40 \log \left| \frac{\mathbf{B}_i(\theta_s, \varphi_s)}{\mathbf{B}_i(\theta, \varphi)} \right| + TS_{theory}^{sph}, \quad (74)$$

$$S_v = 20 \log \left(\frac{r \cdot r_1}{r_s^2} \right) + 2\hat{\alpha}(r-r_s) + 20 \log \left(\frac{\lambda_s}{\lambda} \right) + 20 \log \left(\frac{F_{VV,s}}{F_{VV}} \right) + 10 \log \left(\frac{2r_2}{c_0 \tau_g} \right) + 10 \log \left(\frac{\tau_s}{\tau_p} \right) + 10 \log \left(\frac{[tivs]_R^{st}}{[tivs]_R^{sph}} \right) + 40 \log |\mathbf{B}_i(\theta_s, \varphi_s)| - 10 \log(\psi) + TS_{theory}^{sph}. \quad (75)$$

For formulation C, insertion of Eq. (33) for $G_{0,eff}$ into Eq. (27) for σ_{bs} , use of the parameter A from Eqs. (30) and (35), and assuming $[tivs]_r$ to be constant from calibration to survey operation, leads to Eq. (72). Similarly, insertion of Eq. (22) for G_0 into Eq. (12) for s_v , leads to Eq. (73).

For formulation D, since $G_{0,eff}$ is used as calibration factor in both of formulations C and D, the analysis given above in connection with formulation C for σ_{bs} , applies also to formula-

tion D, leading to Eq. (72). In a similar analysis for s_v , insertion of Eq. (53) for $s_{a,corr}$ into Eq. (51) for s_v , leads to Eq. (73).

For formulation E, the same argumentation with respect to $G_{0,eff}$ as for formulation D, leads to Eq. (72). Similarly, insertion of Eq. (66) for τ_{eff} into Eq. (64) for s_v , leads to Eq. (73).

It follows that the functional relationships for formulations B-E, when the respective calibration factors are inserted in the expressions for σ_{bs} and s_v , are all identical, and given by Eqs. (72)-(73). These equations may thus be referred to as a generic (formulation and instrument independent) echo integration functional relationship for fish abundance estimation, given in terms of the basic quantities being measured in calibration and oceanic surveying.

Eqs. (72)-(73) clearly show which parameters that influence on the oceanic survey measurements, and how an uncertainty or a possible error in one or several of these parameters, influences on the measurement accuracy, - for each of the quantities being measured or assumed known in calibration and oceanic survey operation. Eqs. (72)-(73) are particularly useful for uncertainty or sensitivity studies, such as with respect to effects of sea temperature, or other possible errors or drift in the parameters involved in abundance measurement and species identification.

8. FISH DENSITY AND BIOMASS ESTIMATION

s_v and S_v , as given, respectively, by Eqs. (12) and (21) (formulations B and C), Eqs. (51) and (52) (formulation D), and Eqs. (64) and (65) (formulation E), or equivalently, Eqs. (73) and (75), represent volume backscattering from the thin spherical shell sub-volume V_p of thickness $dr_p = \frac{1}{2}c_0\tau_p$ in the observation volume V_{obs} . The volume backscattering from V_{obs} , between ranges r_{min} and r_{max} , is obtained by measuring s_v for a continuous sequence of "ping volumes", V_p , and integrating s_v over the range of these volumes, giving the area backscattering coefficient of V_{obs} [10,15,14],

$$s_a \equiv \int_{r_{min}}^{r_{max}} s_v(r) dr, \quad (76)$$

representing the backscattering cross section per unit area, within V_{obs} . In echosounder output, s_a (non-dimensional) is frequently given in terms of the nautical area scattering coefficient (NASC) [10,15,5] (m^2),

$$s_A \equiv 4\pi \cdot 1852^2 \cdot s_a, \quad (77)$$

where s_a has been multiplied by the surface area of a sphere with radius one nautical mile. The density of targets (fish), expressed as the (non-dimensional) number of specimens in V_{obs} per square nautical mile, is then given as [15]

$$\rho_a = \frac{s_A}{4\pi \langle \sigma_{bs} \rangle} = \frac{1852^2 \cdot s_a}{\langle \sigma_{bs} \rangle}, \quad (78)$$

where $\langle \sigma_{bs} \rangle$ (m^2) is the expected value of the backscattering cross section (representing the expected TS) of individual targets (fish) [7,9,3,5]. Insertion of Eq. (12) in Eqs. (76)-(78) yields [14]

$$\rho_a = \frac{C_{cal}}{\psi \langle \sigma_{bs} \rangle} E, \quad (79)$$

where

$$E \equiv \int_{r_{min}}^{r_{max}} r^2 e^{4\alpha r} [tivs]_R^{sv} dr, \quad (80)$$

$$C_{cal} \equiv \frac{32\pi^2 \cdot 1852^2}{G_0^2 \cdot \lambda^2 c_0 \cdot \tau_g \cdot F_{VV}^2 \cdot [tivs]_R}. \quad (81)$$

By knowing the expected mass of individual targets, $\langle m \rangle$ (kg), the total biomass in V_{obs} per square nautical mile (kg), may be calculated as

$$m = \rho_a \cdot \langle m \rangle. \quad (82)$$

Eq. (79) is on the form of the traditional echo-integrator equation [7,9,3,5,14], where C_{cal} represents the calibration factor used in that equation. E is the "range integrated echo integral" for the observation volume V_{obs} , obtained by integrating the sequence of echo integrals $[tivs]_R^{sv}$, each associated with a "ping volume" V_p . For each V_p at range r in V_{obs} , the term $r^2 e^{4\alpha r}$ in E is the usual "20log(r) + 2αr" TVG (time-varied gain) factor for volume backscattering from V_p [3]. Similarly, for each V_p , $[tivs]_R^{sv}$ is the echo integral [3] for the "gated volume" V_g , taken over the time interval τ_g , and typically averaged over many transmissions [3], cf. Section 3.1. Since $[tivs]_R^{sv}$ is calculated for each V_p , it depends on range, r .

Eqs. (79)-(81) give the connection between the theory presented here (formulations A-E), and the traditional echo-integrator equation used e.g. by [7,9,3,5]. In that literature, C_{cal} was used solely as an unspecified calibration factor. Here, the functional relationship for C_{cal} is derived, and fully given for small-amplitude conditions in terms of the echosounder system parameters.

9. DISCUSSION

In Sections 2-6, five different but equivalent formulations A-E of the power budget equations and calibration factors for σ_{bs} and s_v , all applicable to fish abundance measurement and spe-

cies identification, have been derived and described. These have been used with the purpose to explain the expressions of, and establish the relationships between, the power budget equations and calibration factors used in some commonly employed scientific and fisheries echosounders and sonar systems.

Appendix A gives an overview of the power budget equations and calibration factors of the five formulations A-E. As discussed in the following, formulation C and D appear to correspond to the expressions used in the EK500 echosounder system; and the EK60, ES60, ME70 and MS70 echosounder and sonar systems, respectively.

9.1 Formulation A (average power, generic)

Formulation A, given by Eqs. (1)-(2) in terms of average electrical power, extends the traditional power budget equations [10] by accounting for arbitrary electrical termination [11,14], represented by an electrical termination factor, F_{Π} .

The expressions given in [10] correspond to setting $F_{\Pi} = 1$ in formulation A, which is valid for electrical termination conditions for which $\mathbf{Z}_E = \mathbf{Z}_R^*$ (conjugate matched electrical termination), or for $\mathbf{Z}_E = \mathbf{Z}_R$ when $X_T = 0$ (i.e., in a frequency band close to the series resonance frequency of the transducer vibration mode used) [14]. Otherwise, formulation A is identical to the expressions given for σ_{bs} and s_v in [10].

As formulation A is not directly suited for signal processing implementation, it serves here as the important fundament for deriving the other four formulations, B-E.

9.2 Formulation B (echo integration, generic)

Formulation B, given in terms of echo integration by Eqs. (11)-(12), together with Eq. (22) for the involved calibration factor G_0 (or equivalently, Eq. (25)), is derived from formulation A to provide expressions better suited for practical signal processing implementation in echosounders [14]. This formulation in terms of echo integrals, for calculation of the electrical powers involved, leads to an alternative electrical termination factor in the power budget equations, F_{VV} . It also involves several integration time intervals, $\tau_p, \tau_{sr}, \tau_s$ and τ_g , to differentiate between integration of the transmitted voltage signal (“ping”); and integration of the single-target (fish), calibration sphere, and volume backscattering voltage echoes, respectively.

Formulation B involves only a single calibration factor, G_0 , which in general may appear advantageous. However, being based on full echo integration both for single target (TS) and volume backscattering measurements, use of this formulation B could possibly be challenging in TS measurement of individual fish at large ranges, in case of poor signal-to-noise ratio. Under such conditions, formulations C-E may be more robust [20], being based on “peak voltage detection” [10] instead of full echo integration, for the TS measurements.

9.3 Formulation C (echo integration, “EK500 type”)

Formulation C, given by Eqs. (27) and (12), together with Eqs. (33) and (22) for the two calibration factors $G_{0,eff}$ and G_0 in-

involved [or equivalently, Eqs. (37) and (44)], appears to correspond to, and generalize, the expressions employed in the Simrad EK500 echosounder [10]. This may be seen as follows.

Firstly, the present theory (cf. Section 4) reveals that the expressions for σ_{bs} and s_v that are actually used in EK500, cannot be exactly those given in its manual [10]. The latter expressions correspond to the average power formulation A (with $F_{\Pi} = 1$), whereas the EK500 implementation [10] involves echo integration, “peak voltage detection” for TS measurements, and two calibration factors “ TS transducer gain” and “ S_v transducer gain”, which neither appear in, nor can be obtained directly from, formulation A.

Secondly, by accounting for echo integration and “peak voltage detection” for TS measurements in formulation C, expressions are here obtained which involve the calibration factors $G_{0,eff}$ and G_0 , given on logarithmic form by Eqs. (38) and (45), respectively. By defining TS transducer gain $\equiv 10\log_{10}(G_{0,eff})$ and S_v transducer gain $\equiv 10\log_{10}(G_0)$, and using Eq. (41), these two equations become identical to the corresponding expressions postulated in the EK500 manual [10]. Formulation C is a prerequisite to obtain these expressions. Hence, formulation C appears to correspond to the equations used in EK500, and the calibration factors “ TS transducer gain” and “ S_v transducer gain” used for EK500, appear to correspond to $G_{0,eff}$ and G_0 , respectively.

The parameter $G_{0,eff}$ has been introduced here to (a) account for “peak voltage detection” in single-target measurement; (b) provide a unifying theory that covers the different formulations A-E of the equations, using a single and consistent terminology; and (c) explain the relationship between the EK500 and EK60 etc. power budget formulations. Note that no such parameter as $G_{0,eff}$ is used in refs. [10,13,16-19].

9.4 Formulation D (echo integration, “EK60 type”)

Formulation D, given by Eqs. (27) and (51), together with Eqs. (33) and (53) for the two calibration factors $G_{0,eff}$ and $s_{a,corr}$ involved [or equivalently, Eqs. (37) and (57) or (59a)], appears to correspond to, and generalize, the expressions used for the Simrad EK60, ES60, ME70 and MS70 systems [13,16-19]. This may be seen as follows.

Firstly, as explained in Section 1.2, the power budget equations given in [10] and [13] are not consistent. This indicates that the expressions for σ_{bs} and s_v which are actually used in EK60 and related instruments, may be different from those given in [13].

Secondly - by accounting for echo integration and “peak voltage detection” for TS measurements as in formulation C – and in addition replacing G_0 with $G_{0,eff} \cdot s_{a,corr}$ [cf. Eq. (50)] in s_v , expressions are obtained that involve the calibration factors $G_{0,eff}$ and $s_{a,corr}$. These calibration factors are given on logarithmic form by Eqs. (38), (58), and (59b). By defining $Gain \equiv 10\log_{10}(G_{0,eff})$ and S_a correction $\equiv S_{a,corr} \equiv 10\log_{10}(s_{a,corr})$,

Eqs. (38) and (59b) become identical to expressions used in EK60 [20]. Moreover, the expressions derived here appear to be consistent with the available information given on the “Gain” and “ S_a correction” calibration factors in the manuals of EK60, etc. [16-19]. Hence, formulation D appears to correspond to the expressions used in the EK60, ES60, ME70 and MS70 systems. The calibration factors “Gain” and “ S_a correction” used for these instruments [16-19], appear to correspond to $G_{0,eff}$ and $s_{a,corr}$, respectively.

The expressions postulated by Ona *et al.* [13] for these instruments, correspond to Formulation D. There are however some deviations. Firstly (in terms of the terminology used here), in [13], electrical powers Π_T and Π_R (in units of W) were used instead of echo integrals, $[tivs]_T$, $[tivs]_R^{sv}$, and $[tivs]_{R,max}^{sv}$ (in units of $V^2s \equiv W\Omega s$). Echo integration was thus not taken into account. Secondly, the same Π_R was used for σ_{bs} and s_v , which does not appear to be consistent with use of “peak voltage detection” and full echo integration, respectively, for TS and s_v (and thus s_A) measurements. Thirdly, $G(\theta, \varphi)$ and G_0 were used instead of $G_{eff}(\theta, \varphi)$ and $G_{0,eff}$ in the expressions for σ_{bs} and s_v , respectively, which does not seem to be consistent, as long as “peak voltage detection” is used for σ_{bs} , and $s_{a,corr}$ is used in s_v . Finally, in [13], electrical termination was not accounted for, implicitly implying $F_{\Pi} = 1$ [11,14], cf. Section 9.1.

In other words, for power budget equations expressed in terms of echo integrals, and for which “peak voltage detection” is used in TS measurements instead of full echo integration, the parameters $G(\theta, \varphi)$ and G_0 , referred to in [13] as “transducer gain” and “on-axis transducer gain”, respectively, are *not* equal to the transducer gain and axial transducer gain, $G(\theta, \varphi)$ and G_0 , that are involved in the traditional power budget equation given in [10] [and in Eqs. (1)-(2) and (11)-(12)], as implicitly stated in [13]. Instead, they represent “effective” transducer and axial transducer gains, respectively, $G_{eff}(\theta, \varphi)$ and $G_{0,eff}$, with a correction factor $s_{a,corr}$ involved, to compensate for the use of “peak voltage detection” in TS measurements instead of full echo integration, cf. Eqs. (27) and (49)-(51).

Through the derivation of formulation D, expressions have been obtained which (a) seem to explain and resolve the deviation between the power budget equations given in [13] and [10], (b) explain the introduction and use of the $S_{a,corr}$ parameter, and (c) “harmonize the TS and s_A measurements” [17] by employing the same calibration factor $G_{0,eff}$ in σ_{bs} and s_v . The power budget expressions that are consistent with [10], and which may replace those given in [13], appear to be Eqs. (27) and (51) [or equivalently, Eqs. (28) and (52)]. This includes calibration factors $G_{0,eff}$ and $s_{a,corr}$, given by Eqs. (33)-(34) [or equivalently Eqs. (37)-(38)], and Eqs. (53)-(54) [or equivalently Eqs. (57)-(58), or Eqs. (59)], respectively.

Through the present analysis, the relationship between the formulation D calibration factors $G_{0,eff}$ and $s_{a,corr}$, and the formulation C calibration factors, $G_{0,eff}$ and G_0 , is established, cf. Eq. (50). This includes their relationships to the generic formulation A and B types of description. It follows that the relationships between the EK500 calibration factors “ TS transducer gain” and “ S_v transducer gain”, and the EK60 etc. calibration factors “Gain” and “ S_a correction”, also appear to have been explained and established.

9.5 Formulation E (echo integration)

Formulation E, given by Eqs. (27) and (63), together with Eqs. (33) and (66) for the two calibration factors $G_{0,eff}$ and τ_{eff} involved [or equivalently, Eqs. (37) and (69) or (62)], represents an alternative and valid formulation for abundance measurement [13].

The interpretation of τ_{eff} as an “effective pulse duration”, as indicated by ref. [13], may however be discussed. In Sections 4.1 and 5.1, $s_{a,corr}$, used to define τ_{eff} in Eq. (60), is shown to be essentially a correction factor to compensate for incorrect echo-integration of the received voltage signal in single-target measurements (e.g., calibration), caused by the use of $[tivs]_{R,max}^{sph}$ instead of $[tivs]_R^{sph}$, for single targets (“peak voltage detection”). Hence, from Eq. (63) it appears that τ_{eff} represents a scaled *gate opening time*, to compensate for erroneous echo integration at *signal reception*, e.g. in the calibration situation. A more correct interpretation of τ_{eff} might thus be “effective echo integration time”.

Without an explanation of these mechanisms, there is the possibility that the use of an interpretation and terminology where τ_{eff} is (erroneously) associated with an effective duration of the transmitted pulse, instead of an effective echo integration time at signal reception, may contribute to confusion among users. To prevent uncertainty on this point, the apparent reason for the introduction and use of $s_{a,corr}$, as revealed by the present analysis, is discussed in the following.

9.6 On the $S_{a,corr}$ parameter

From the analysis, the reason for using two calibration factors in the EK500 echosounder [10], two different calibration factors in EK60 and related systems [13,16-19], and to introduce $S_{a,corr}$ in the expression for s_v of the latter instruments [13], appears to have been the following.

If the generic formulation B of the power budget equations was used, only a single calibration factor G_0 would be necessary (cf. Section 9.2).

However, due to the use of “peak voltage detection” instead of full echo integration in TS measurements (i.e., use of $[tivs]_{R,max}^{sph}$ instead of $[tivs]_R^{sph}$ for single targets), an error is introduced in σ_{bs} , which is here compensated for by introducing $G_{0,eff}$ instead of G_0 in σ_{bs} , cf. Eq. (27). By still using G_0 in the

expression for s_v , cf. Eq. (12), two calibration factors $G_{0,eff}$ and G_0 thus become involved in the EK500 implementation, referred to as “TS transducer gain” and “ S_v transducer gain”, respectively [10].

In the more recent generation of Simrad echosounder and sonar systems, EK60, ES60, ME70 and ME70 [13,16-19], the calibration factor G_0 used for EK500 (in the s_v expression), appears to have been replaced by $G_{0,eff}$, to “harmonize” the σ_{bs} and s_v measurements [17]. An error is then introduced in s_v , which is compensated for by using $s_{a,corr}$ in s_v , referred to as the “ S_a correction”, cf. Eqs. (51)-(52).

Unfortunately, these aspects do not appear to have been explained or communicated in available literature, including [10,13,16-19]. In ref. [13], $G(\theta, \varphi)$ and G_0 are incorrectly used instead of $G_{eff}(\theta, \varphi)$ and $G_{0,eff}$, in the expressions for σ_{bs} and s_v (cf. Section 9.4).

9.7 Comments in relation to conventional operation

It is emphasized that the derivation and presentation of formulations B-E by no means indicates that any new method for abundance estimation is proposed, as an alternative to the conventional method used today. The situation is quite the opposite: As explained in Section 1 and above, the expressions presented here are intended to provide a consistent and unifying theory for improved understanding and control in use of the conventional method, employing these commonly employed echosounder and sonar systems.

In addition, the more general and complete expressions derived here, constitute an improved basis for evaluation and, if necessary, correction of errors in abundance estimation and species determination, such as due to possible drift due to e.g. changing environment or echosounder parameters, from calibration to oceanic surveying. Such possibilities are limited as long as the full functional relationship for the abundance measurement, in terms of echosounder and environmental parameters, is not known for the echosounder system in question.

10. CONCLUSIONS

Acoustic methods for fish abundance estimation and species identification rely on power budget equations and calibrated echosounder and sonar systems. Different instrument specific formulations of power budget equations and calibration factors are used in some commonly employed instruments, such as the Simrad EK500, EK60, ES60, ME70 and MS70 systems [10,13,16-19].

It appears that the documentation in prior literature on the actual equations and calibration factors employed in these instruments, may be somewhat insufficient. Some of the expressions specified [10,13] are not readily derivable from the literature in this field [1,3-5,9-13]. Moreover, the power budget equations given for the Simrad EK60, ES60, ME70 and MS70 systems [13] do not appear to be consistent with the literature on the Simrad EK500 system [10]. The latter represents an im-

portant fundament for today’s methods in fish abundance measurement.

This situation - together with inconsistencies in prior literature on these instruments - has caused some uncertainty and confusion among users. For improved control and understanding in use of these systems, a documentation of the expressions employed, and their relationship to the traditional theory of fishery acoustics [10], is addressed. In particular for analysis of - and possible correction for - measurement errors due to system drift from calibration to oceanic surveying, such control is essential.

The paper presents a unifying theory which seems to explain the different power budget equation formulations and calibration factors employed in the mentioned echosounder and sonar systems. This includes how they are mutually related, as well as their relationship to the traditional and generic (instrument independent) theory of fish abundance measurement. Inconsistencies in prior literature on these systems appear to have been explained and corrected.

In addition, for improved control with systematic errors and system drift, earlier literature on the underlying theory in this field [1,3-5,9-13] is extended to provide more complete power budget equations for fish abundance estimation and species identification. This includes arbitrary electrical termination, representation of echo integration, and the full range of echosounder parameters involved in calibration and oceanic surveying. These extensions apply to the instrument specific as well as instrument independent formulations of these equations.

For these purposes, five different but equivalent formulations of the power budget equations for σ_{bs} and s_v , and the associated calibration factors, are presented. Generic power budget equations expressed in terms of average electrical power (denoted formulation A), are used as a fundament for deriving (1) corresponding generic power budget equations expressed in terms of full echo integration (denoted formulation B), and (2) three alternative sets of equivalent, but more instrument specific, power budget equations (denoted formulations C, D and E).

It is shown that the echo integration formulations C and D derived here, are closely related to the expressions and calibration factors used in the Simrad EK500 echosounder [10], and the more recent generation of Simrad echosounders and sonar systems, EK60, ES60, ME70 and MS70 [13,16-19], respectively. Formulations C and D also extend the power budget equations corresponding to these echosounders, to provide more complete expressions, as described above.

The analysis shows that establishment of the generic formulation B, expressed in terms of echo integration [14], is the key to derive formulations C-E, and thus to explain the formulations corresponding to EK500 and EK60 etc. Formal representation of echo integration, as derived in [14], is essential to derive and understand these equations.

Formulations C-E are shown to originate from the use of “peak voltage detection” in single-target (TS) measurements [10] instead of full echo integration. A parameter $G_{0,eff}$ is introduced here to account for such “peak voltage detection”; to avoid inconsistencies in terminology (cf. Section 9.4); to clarify

the relationships between the various formulations A-E; and to provide full consistency with the terminology of the traditional power budget equations given in [10]. In particular, it is shown that for EK500 [10] and for EK60 and related systems [13,16-19], the “TS transducer gain” and “Gain” calibration factors, respectively, are not related to the axial transducer gain, G_0 (as it may appear from [10,13,17]), but to the parameter $G_{0,eff}$.

Relationships between the equivalent formulations A-E are derived, and between the different calibration factors involved; G_0 (formulation B), $G_{0,eff}$ and G_0 (formulation C), $G_{0,eff}$ and $s_{a,corr}$ (formulation D); and $G_{0,eff}$ and τ_{eff} (formulation E), cf. Eqs. (50), (60), and (62). As a result, the mathematical relationships appear to have been established between the power budget equations and calibration factors employed in EK500 [10], and the more recent generation of echosounder and sonar systems, EK60, ES60, ME70 and MS70 [13,16-19]. This includes establishment of their relationship to the generic formulations A and B [14], and thus to the traditional (but less general) power budget equations given in the EK500 manual [10].

The lack of literature documentation of the power budget equations and calibration factors actually used in the EK500, EK60, ES60, ME70 and MS70 systems, unfortunately prevents a complete comparison of the expressions derived here for formulations C and D, with those used in these echosounders. The expressions given in available literature for EK500 [10] are however in agreement with the theory presented here (cf. Section 9). The consistency obtained with respect to the calibration factors “TS transducer gain” and “S_v transducer gain”, cf. Eqs. (38) and (45), strongly indicates correspondence between the implementation used in EK500, and formulation C. For the EK60 and related systems, less theory and equations are available. However, the consistency obtained in relation to the power budget equations given in [13] (after correction of those expressions, cf. Sections 5 and 9.4, and Appendix A.4), and the information given on calibration factors in relevant manuals and correspondence [16-20], strongly indicates correspondence between the implementation used in EK60 etc., and formulation D.

The functional relationships A-E, including their respective calibration factors, are here fully given in terms of the echosounder parameters being measured or assumed known in calibration and oceanic surveying. This includes generic instrument independent formulations, as well as more instrument specific formulations, applicable to the Simrad EK500, EK60, ES60, ME70 and MS70 echosounder and sonar systems.

Under assumption of small-amplitude (linear) sound propagation, the expressions derived are expected to represent a consistent and relatively complete theoretical basis for improved understanding and control, in use of conventional methods and instruments for fish abundance measurement and species identification. In addition to establishing a unified theory for use of different equipment in such applications, the results are expected to constitute an improved theoretical fundament for measurement, error evaluation, possible error compensation, and uncertainty evaluation, of fish abundance methods and equipment in use today.

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APPENDIX A. SUMMARY OF POWER BUDGET EQUATION FORMULATIONS

For convenience, an overview of the power budget equations and calibration factors that are involved in formulations A-E, is given in the following. All formulations A-E are equivalent (and also equivalent to the generic formulation given in Section 7, which is not shown here). For each formulation A-E, two or three equivalent and alternative calculation methods for the calibration factors are included (denoted methods 1, 2 and 3).

“Method 1” gives the calibration factors directly in terms of the basic echosounder parameters that are measured at calibration.

“Method 2” gives the calibration factors on forms that are similar to the forms used in the EK500 manual [10], for the two calibration factors of the EK500 system, cf. Eqs. (38) and (45).

“Method 3” (used for formulations D and E) gives one of the calibration factors on an alternative form which may be of interest if an EK60 type echosounder (or similar) is operated more as an EK500 type echosounder.

Methods 2 and 3 are given on normal and logarithmic forms (since logarithmic expressions are used in [10]). Equation numbers from the main text are used. All involved parameters are defined and given in the main text.

A.1 Formulation A (average power, generic)

Power budget equations:

$$\sigma_{bs} = \frac{16\pi^2 \cdot r^4 \cdot e^{4\alpha r} \cdot \Pi_R^s}{G^2(\theta, \varphi) \cdot \lambda^2 \cdot F_{\Pi} \cdot \Pi_T} \quad (1)$$

$$s_v = \frac{32\pi^2 \cdot r^2 \cdot e^{4\alpha r} \cdot \Pi_R^v}{G_0^2 \cdot \psi \cdot \lambda^2 c_0 \cdot \tau_p \cdot F_{\Pi} \cdot \Pi_T} \quad (2)$$

A.2 Formulation B (echo integration, generic)

Power budget equations:

$$\sigma_{bs} = \frac{16\pi^2 \cdot r^4 \cdot e^{4\alpha r} \cdot \tau_p \cdot [tivs]_R^s}{G^2(\theta, \varphi) \cdot \lambda^2 \cdot \tau_{st} \cdot F_{VV}^2 \cdot [tivs]_T} \quad (11)$$

$$s_v = \frac{32\pi^2 \cdot r^2 \cdot e^{4\alpha r} \cdot [tivs]_R^{sv}}{G_0^2 \cdot \psi \cdot \lambda^2 c_0 \cdot \tau_g \cdot F_{VV}^2 \cdot [tivs]_T} \quad (12)$$

Calibration factor G_0 (equivalent expressions):

Method 1:

$$G_0^2 = \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot \tau_p \cdot [tivs]_R^{sph}}{\lambda_s^2 \cdot F_{VV,s}^2 \cdot \tau_s \cdot \sigma_{bs,theory}^{sph} \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4 \cdot [tivs]_T} \quad (22)$$

Method 2:

$$G_0^2 = G_{0,old}^2 \cdot \frac{\sigma_{bs,old}^{sph}}{\sigma_{bs,theory}^{sph}} \quad (25)$$

$$10\log(G_0) = 10\log(G_{0,old}) + \frac{1}{2} \cdot [TS_{old}^{sph} - TS_{theory}^{sph}] \quad (26)$$

A.3 Formulation C (echo integration, “EK500 type”)

Power budget equations:

$$\sigma_{bs} = \frac{16\pi^2 \cdot r^4 \cdot e^{4\alpha r} \cdot [tivs]_{R,max}^{st}}{G_{eff}^2(\theta, \varphi) \cdot \lambda^2 \cdot F_{VV}^2 \cdot [tivs]_T} \quad (27)$$

$$s_v = \frac{32\pi^2 \cdot r^2 \cdot e^{4\alpha r} \cdot [tivs]_R^{sv}}{G_0^2 \cdot \psi \cdot \lambda^2 \cdot c_0 \cdot \tau_g \cdot F_{VV}^2 \cdot [tivs]_T} \quad (12)$$

Calibration factor $G_{0,eff}$, “TS transducer gain” (equivalent expressions):

Method 1:

$$G_{0,eff}^2 \equiv \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot [tivs]_{R,max}^{sph}}{\lambda_s^2 \cdot \sigma_{bs,theory}^{sph} \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4 \cdot F_{VV,s}^2 \cdot [tivs]_T} \quad (33)$$

Method 2:

$$G_{0,eff}^2 = G_{0,eff,old}^2 \cdot \frac{\sigma_{bs,old}^{sph}}{\sigma_{bs,theory}^{sph}} \quad (37)$$

$$10\log(G_{0,eff}) = 10\log(G_{0,eff,old}) + \frac{1}{2} \cdot [TS_{eff,old}^{sph} - TS_{theory}^{sph}] \quad (38)$$

Calibration factor G_0 , “ S_V transducer gain” (equivalent expressions):

Method 1:

$$G_0^2 = \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot \tau_p \cdot [tivs]_R^{sph}}{\lambda_s^2 \cdot F_{VV,s}^2 \cdot \tau_s \cdot \sigma_{bs,theory}^{sph} \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4 \cdot [tivs]_T} \quad (22)$$

Method 2:

$$G_0^2 = G_{0,old}^2 \cdot \frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}} \quad (44)$$

$$10\log(G_0) = 10\log(G_{0,old}) + \frac{1}{2} \cdot 10\log\left(\frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}}\right) \quad (45)$$

$$\text{with } s_{a,theory}^{sph} = \frac{\tau_s}{\tau_p} \cdot \frac{|\mathbf{B}_i(\theta_s, \varphi_s)|^4}{\psi \cdot r_s^2} \cdot \sigma_{bs,theory}^{sph} \quad (41)$$

A.4 Formulation D (echo integration, “EK60 type”)

Power budget equations:

$$\sigma_{bs} = \frac{16\pi^2 \cdot r^4 \cdot e^{4\alpha r} \cdot [tivs]_{R,max}^{sv}}{G_{eff}^2(\theta, \varphi) \cdot \lambda^2 \cdot F_{VV}^2 \cdot [tivs]_T} \quad (27)$$

$$s_v = \frac{32\pi^2 \cdot r^2 \cdot e^{4\alpha r} \cdot [tivs]_R^{sv}}{G_{0,eff}^2 \cdot \psi \cdot \lambda^2 \cdot c_0 \cdot \tau_g \cdot s_{a,corr}^2 \cdot F_{VV}^2 \cdot [tivs]_T} \quad (51)$$

Calibration factor $G_{0,eff}$, “Gain” (equivalent expressions):

Method 1:

$$G_{0,eff}^2 \equiv \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot [tivs]_{R,max}^{sph}}{\lambda_s^2 \cdot \sigma_{bs,theory}^{sph} \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4 \cdot F_{VV,s}^2 \cdot [tivs]_T} \quad (33)$$

Method 2:

$$G_{0,eff}^2 = G_{0,eff,old}^2 \cdot \frac{\sigma_{bs,old}^{sph}}{\sigma_{bs,theory}^{sph}} \quad (37)$$

$$10\log(G_{0,eff}) = 10\log(G_{0,eff,old}) + \frac{1}{2} \cdot [TS_{eff,old}^{sph} - TS_{theory}^{sph}] \quad (38)$$

Calibration factor $s_{a,corr}$, “ S_a correction” (equivalent expressions):

Method 1:

$$s_{a,corr}^2 = \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot \tau_p \cdot [tivs]_R^{sph}}{G_{0,eff}^2 \cdot \lambda_s^2 \cdot \tau_s \cdot \sigma_{bs,theory}^{sph} \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4 \cdot F_{VV,s}^2 \cdot [tivs]_T} \quad (53)$$

Method 2:

$$s_{a,corr}^2 = s_{a,corr,old}^2 \cdot \frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}} \quad (57)$$

$$S_{a,corr} = S_{a,corr,old} + \frac{1}{2} \cdot 10\log\left(\frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}}\right) \quad (58)$$

$$\text{with } s_{a,theory}^{sph} = \frac{\tau_s}{\tau_p} \cdot \frac{|\mathbf{B}_i(\theta_s, \varphi_s)|^4}{\psi \cdot r_s^2} \cdot \sigma_{bs,theory}^{sph} \quad (41)$$

Method 3:

$$s_{a,corr} = \frac{G_0}{G_{0,eff}}, \quad (59a)$$

$$S_{a,corr} = 10\log(G_0) - 10\log(G_{0,eff}). \quad (59b)$$

A.5 Formulation E (echo integration)

Power budget equations:

$$\sigma_{bs} = \frac{16\pi^2 \cdot r^4 \cdot e^{4\alpha r} \cdot [tivs]_{R,max}^{sv}}{G_{eff}^2(\theta, \varphi) \cdot \lambda^2 \cdot F_{VV}^2 \cdot [tivs]_T} \quad (27)$$

$$s_v = \frac{32\pi^2 \cdot r^2 \cdot e^{4\alpha r} \cdot [tivs]_R^{sv}}{G_{0,eff}^2 \cdot \psi \cdot \lambda^2 \cdot c_0 \cdot \tau_{eff} \cdot F_{VV}^2 \cdot [tivs]_T} \quad (64)$$

Calibration factor $G_{0,eff}$, “Gain” (equivalent expressions):

Method 1:

$$G_{0,eff}^2 \equiv \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot [tivs]_{R,max}^{sph}}{\lambda_s^2 \cdot \sigma_{bs,theory}^{sph} \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4 \cdot F_{VV,s}^2 \cdot [tivs]_r} \quad (33)$$

Method 2:

$$G_{0,eff}^2 = G_{0,eff,old}^2 \cdot \frac{\sigma_{bs,old}^{sph}}{\sigma_{bs,theory}^{sph}} \quad (37)$$

$$10\log(G_{0,eff}) = 10\log(G_{0,eff,old}) + \frac{1}{2} \cdot [TS_{eff,old}^{sph} - TS_{theory}^{sph}] \quad (38)$$

Calibration factor τ_{eff} , “Effective echo integration time” (equivalent expressions):

Method 1:

$$\tau_{eff} = \frac{16\pi^2 \cdot r_s^4 \cdot e^{4\alpha r_s} \cdot \tau_p \cdot [tivs]_R^{sph}}{G_{0,eff}^2 \cdot \lambda_s^2 \cdot F_{VV,s}^2 \cdot [tivs]_r \cdot \sigma_{bs,theory}^{sph} \cdot |\mathbf{B}_i(\theta_s, \varphi_s)|^4} \quad (66)$$

Method 2:

$$\tau_{eff} = \tau_{eff,old} \cdot \frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}} \quad (69)$$

$$10\log\left(\frac{\tau_{eff}}{\tau_{ref}}\right) = 10\log\left(\frac{\tau_{eff,old}}{\tau_{ref}}\right) + 10\log\left(\frac{s_{a,old}^{sph}}{s_{a,theory}^{sph}}\right) \quad (70)$$

$$\text{with } s_{a,theory}^{sph} = \frac{\tau_s}{\tau_p} \cdot \frac{|\mathbf{B}_i(\theta_s, \varphi_s)|^4}{\psi \cdot r_s^2} \cdot \sigma_{bs,theory}^{sph} \quad (41)$$

Method 3:

$$\tau_{eff} = \tau_g \cdot \frac{G_0^2}{G_{0,eff}^2}, \quad (62)$$

$$10\log\left(\frac{\tau_{eff}}{\tau_{ref}}\right) = 10\log\left(\frac{\tau_g}{\tau_{ref}}\right) + 2[10\log(G_0) - 10\log(G_{0,eff})] \quad (71)$$

APPENDIX B. INTERPRETATION IN TERMS OF POWER FLOW

B.1 Single-target backscattering

For physical interpretation of the various terms in the power budget equation describing single-target backscattering, Eq. (1), the following re-arrangement serves convenient,

$$\Pi_R^{st} = \Pi_T \cdot G(\theta, \varphi) \cdot \frac{e^{-2\alpha r}}{4\pi r^2} \cdot \sigma_{sp} \cdot \frac{e^{-2\alpha r}}{4\pi r^2} \cdot G(\theta, \varphi) \cdot \frac{\lambda^2}{4\pi} \cdot F_{\Pi} \quad (B.1)$$

Here, $\sigma_{sp} = 4\pi\sigma_{bs}$ is the spherical scattering cross-section [1] for the equivalent omnidirectional scatterer of the single target. Π_T is the average transmitted electrical power, averaged over one cycle of the harmonic wave, at the frequency f in question. Multiplying with $G(\theta, \varphi)$ gives the acoustic power produced by a point source radiating an (omnidirectional) intensity that is equal to the transducer’s radiated intensity in

(θ, φ) direction at range r , under lossless and small-amplitude sound propagation conditions in the fluid. Multiplying with $e^{-2\alpha r}/4\pi r^2$ yields the transducer’s radiated intensity at the target position, (r, θ, φ) , at small-amplitude conditions, and absorption accounted for. Multiplication with σ_{sp} gives the acoustic power scattered by the target, here represented by the target’s equivalent omnidirectional scatterer. Multiplying with $e^{-2\alpha r}/4\pi r^2$ yields the free-field acoustic power density of the scattered field at the centre of the transducer front, with absorption accounted for. Multiplication with the “effective area” (or “effective aperture”) [10,25,27] of the receiving transducer, $G(\theta, \varphi) \cdot (\lambda^2/4\pi)$, yields the received electrical power at the transducer’s electrical terminals, for the particular electrical termination case $F_{\Pi} = 1$ [i.e., either for $\mathbf{Z}_E = \mathbf{Z}_R^*$ (conjugate matched electrical termination, to maximise the power transfer to the receiving electronic circuit); or $\mathbf{Z}_E = \mathbf{Z}_R$ when $X_T = 0$ (to minimize signal reflections from the electrical load, in the frequency band of the transducer’s series resonance)] [14]. Finally, multiplying with $F_{\Pi} = 4R_T R_E / |\mathbf{Z}_R + \mathbf{Z}_E|^2$ yields the average received electrical power Π_R^{st} at the transducer’s electrical terminals, for arbitrary electrical termination load [27].

B.2 Volume backscattering

Similarly, physical interpretation of the various terms in the power budget equation describing volume backscattering from V_p , Eq. (2), is conveniently made using the re-arrangement

$$\Pi_R^v = \Pi_T \cdot G_0 \cdot \frac{e^{-2\alpha r}}{4\pi r^2} \cdot \sigma_{sp}^v \cdot \frac{e^{-2\alpha r}}{4\pi r^2} \cdot G_0 \cdot \frac{\lambda^2}{4\pi} \cdot F_{\Pi}, \quad (B.2)$$

where $V_p \approx 4\pi r_p^2 (r_{p2} - r_{p1}) = 4\pi r_p^2 dr_p = 4\pi r_p^2 \cdot \frac{1}{2} c_0 \tau_p$, and the definition

$$\sigma_{sp}^v \equiv s_v \cdot V_p \cdot \psi, \quad (B.3)$$

has been used.

First, an interpretation of the quantity σ_{sp}^v serves to be useful. The equivalent two-way beam solid angle, ψ , represents the transducer’s effective beam width (in terms of a solid angle) for transmission and reception combined. From the definition of a solid angle, the portion of the surface area of a sphere with radius at r_p that is effectively insonified by the equivalent two-way beam solid angle, equals $A_s \equiv r_p^2 \cdot \psi$, and is here denoted the “sampled area”. Consequently, $V_p \cdot \psi \approx 4\pi r_p^2 \cdot dr_p \cdot \psi = 4\pi \cdot V_s$, where $V_s \equiv dr_p \cdot A_s$ is denoted the “sampled volume” portion of the spherical shell volume, V_p , that is contained within the range interval $[r_{p1}, r_{p2}]$ and the solid angle ψ . Hence, V_s represents that portion of the assumed homogeneous distribution of omnidirectional scattering

targets contained in V_p , that is effectively insonified by the acoustic beam, upon radiation and reception combined. It follows that $V_s \approx V_p \cdot \psi / 4\pi$. Now, for volume backscattering, one has, for the assumptions stated in Section 2.1, $d\sigma_{bs} = s_v \cdot dV$, where $d\sigma_{bs}$ represents the backscattering cross section of a multitude of objects (targets) in a unit volume dV in V_p , including objects of different types, and objects of the same type with different sizes [14]. From Eq. (B.3) it follows that $\sigma_{sp}^v = (d\sigma_{sp} / dV) \cdot V_s$, where $d\sigma_{sp} = 4\pi \cdot d\sigma_{bs}$ represents the spherical scattering cross section of the scatterers in dV . Consequently, σ_{sp}^v represents the equivalent spherical scattering cross section of the sampled volume, V_s . The corresponding equivalent backscattering cross section of V_s is given as $\sigma_{bs}^v \equiv \sigma_{sp}^v / 4\pi = s_v \cdot V_p \cdot \psi / 4\pi$.

In Eq. (B2), thus, by following the reasoning used for interpretation of Eq. (B1), $\Pi_T \cdot G_0 \cdot (e^{-2\alpha} / 4\pi r^2)$ gives the transducer's radiated intensity in axial direction, at the V_p range, $r \approx r_p$, with absorption accounted for. Multiplication with the equivalent spherical scattering cross section of the sampled volume, σ_{sp}^v , gives the acoustic power scattered from V_p , here represented by the sampled volume's omnidirectional scatterer. Multiplying with $e^{-2\alpha} / 4\pi r^2$ yields the free-field acoustic power density of the scattered field at the center of the transducer front. Multiplication with the "effective area" [25,27] of the receiving transducer, for normally incident sound to the transducer ($\theta = \varphi = 0$), $G_0 \cdot (\lambda^2 / 4\pi)$, yields the received electrical power at the transducer's electrical terminals, for the special case of $F_{\Pi} = 1$ [i.e., either for $\mathbf{Z}_E = \mathbf{Z}_R^*$ (conjugate matched electrical termination, to maximise the power transfer to the receiving electronic circuit); or $\mathbf{Z}_E = \mathbf{Z}_R$ when $X_T = 0$ (to minimize signal reflections from the electrical load, in the frequency band of the transducer's series resonance)] [14]. Finally, multiplying with $F_{\Pi} = 4R_T R_E / |\mathbf{Z}_R + \mathbf{Z}_E|^2$ yields the average received electrical power Π_R^v at the transducer's electrical terminals, for arbitrary electrical termination load [27].

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